Computational Statistics and Data Analysis 53 (2009) 2251–2263

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

A robust forward weighted Lagrange multiplier test for conditional heteroscedasticity

Luigi Grossi[a,*], Fabrizio Laurini[b]

[a] *Department of Economics, University of Verona, Italy*

[b] *Department of Economics, University of Parma, Italy*

ARTICLE INFO

Article history:
Available online 25 September 2008

ABSTRACT

Statistical tests routinely adopted for detecting nonlinear components in time series rely on the auxiliary regression of ARMA lagged residuals, and the Lagrange multiplier test to detect ARCH components is an example. The size distortion of such test suggests adopting a weighted test, where the weights are computed through a forward search algorithm. Simulations show that the forward weighted robust test is preferable to the classical Lagrange test and to existing robust tests, which are based on backward weighted regression or on estimated autocorrelation function. The forward weighted robust test is applied to daily financial and quarterly macroeconomic time series, showing its usefulness in detecting ARCH effects, even when outliers are present.

1. Introduction

The generalized autoregressive conditionally heteroscedastic (GARCH) processes, introduced by Bollerslev (1986), easily extend to ARMA-GARCH processes, when the conditional mean of a time series r_t is time-varying. The ARMA(s, v)–GARCH(p, q) model can be written as

$$r_t = \kappa + \sum_{\iota=1}^{s} \phi_\iota r_{t-\iota} + \sum_{l=1}^{v} \vartheta_l \eta_{t-l} + \eta_t,$$

$$\eta_t = a_t \sqrt{h_t}$$

and

$$h_t = \omega + \sum_{i=1}^{q} \lambda_i \eta_{t-i}^2 + \sum_{j=1}^{p} \gamma_j h_{t-j},$$

where $\{a_t\}$ is a zero-mean unit-variance independent identically distributed (*iid*) sequence, typically Gaussian. All ARMA parameters $\kappa, \phi_\iota, \iota = 1, \ldots, s, \vartheta_l, l = 1, \ldots, v$ and GARCH parameters $\omega > 0, \lambda_i \geq 0, i = 1, \ldots, q$ and $\gamma_j \geq 0, j = 1, \ldots, p$ are assumed to satisfy constraints for stationarity. Special cases are represented by $s = v = 0$, which gives GARCH(p, q) or $\{s = v = p = 0, q = 1\}$, which is an ARCH(1).

Engle (1982) derived a test for the identification of ARCH components, based on Lagrange multipliers (LM), through an auxiliary regression of residuals of a conditional mean fit $\hat{\eta}_t = r_t - E_r$, where E_r is an ARMA fit for the conditional mean of the process r_t. The auxiliary regression is then:

$$\hat{\eta}_t^2 = \alpha_0 + \alpha_1 \hat{\eta}_{t-1}^2 + \cdots + \alpha_q \hat{\eta}_{t-q}^2 + \varepsilon_t, \quad t = q+1, \ldots, T, \tag{1}$$

* Corresponding address: Department of Economics, University of Verona, Via dell'Artigliere 8, 37129, Verona, Italy. Tel.: +39 0458028007.
E-mail addresses: luigi.grossi@univr.it (L. Grossi), fabrizio.laurini@unipr.it (F. Laurini).

0167-9473/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.csda.2008.09.018

where ε_t is the innovation of the auxiliary regression, independent of a_t. The null hypothesis of conditional homoscedasticity can be formulated as $H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$. This way of testing for ARCH components can be computed using $LM = TR^2$, where R^2 is the squared multiple correlation coefficient of model (1). Under the null hypothesis, the LM test statistic has an asymptotic $\chi^2(q)$ distribution. Lee (1991) showed that the LM test against a GARCH(p, q) alternative is the same as the LM test against the alternative of an ARCH(q), i.e. the auxiliary regression (1) can be used to detect GARCH effects.

It is possible to test for other effects (e.g. the leverage) following the same strategy by suitably modifying the auxiliary regression (1), for which the statistic TR^2 still has an asymptotic χ^2 distribution. Examples are the quadratic GARCH (QGARCH) of Sentana (1995), the threshold model proposed by Glosten et al. (1993), the exponential smooth transition GARCH (ESTARCH) and logistic smooth transition GARCH (LSTARCH) proposed by Hagerud (1997) and by Gonzalez-Rivera (1998), respectively.

It is well known that the LM test is sensitive to outliers, leading to two different wrong conclusions. Van Dijk et al. (1999) show that, in some cases, outliers lead the LM test to reject the null hypothesis of conditional homoscedasticity too often and, in other cases, outliers may hide true heteroscedasticity. These findings motivated the use of a weighted regression LM test (Van Dijk et al., 1999), studied also by Carnero et al. (2007), that exhibits size distortion in large samples. A different approach involves the robust estimation of autocorrelations of the squared residuals (Duchesne, 2004), combined with the one-sided test for ARCH effects developed by Hong (1997).

The general effect of outliers on LM tests are discussed in Tolvi (2000) who studied size distortion and power of the Van Dijk et al. (1999) test. The severity of the size distortion depends on the type of outlier, its magnitude and on the underlying data generating process. Similar conclusions are obtained by Balke and Fomby (1994), Aggarwal et al. (1999) and Franses et al. (2004), in the context of financial and macroeconomic time series.

It is less known that the LM test can identify many ARCH effects even when the true generating process is an ARCH(1) or a Gaussian *iid* sequence (see Grossi and Laurini (2004)). The robust test of Van Dijk et al. (1999) has low power when data are not contaminated, as it may wrongly detect outliers when the generating process is a Gaussian *iid* sequence.

In this paper we address the problems of size distortion and power, proposing a new way of weighting observations for the robust LM test. The robust LM test, computed with the new system of weights, has lower size distortion than both the LM test and the test of Van Dijk et al. (1999), and also has higher power than the test of Van Dijk et al. (1999).

Our test exploits the new approach to diagnostic regression, and outlier detection, proposed by Atkinson and Riani (2000), through the forward search method. A forward analysis of the LM test for GARCH components has been proposed by Grossi and Laurini (2004), who suggest taking some care in drawing inferential conclusions when t statistics and R^2 tests are used to assess the presence of non-linear components. However, Grossi and Laurini (2004) did not address the issue of using the forward search to improve the size and the power of the LM test, either when outliers are present or when data are outlier-free.

Here, we fill the gap introducing a new method to compute the weights with the forward search algorithm, and use these weights in the auxiliary regression of the LM test. The weights take account of the "degree of outlyingness" of each observation, exploiting the capability of the forward search to reveal complex structure of data even when contaminated by patches of outliers (see Atkinson et al. (2004) and the references therein).

The paper is organized as follows. Section 2 reviews the forward search steps and introduces a method for weighting observations in time series. In Section 3 we present some results on simulated data, while, in Section 4, we study real time series. We close with some final remarks and discussion.

2. Robust LM test through the forward search

2.1. General comments

Our target is to compute weights $w_t \in [0, 1]$, for each observation in the time series y_t, $t = 1, \ldots, T$, with the forward search method. The weights will be used in a weighted linear regression, and they are such that the most outlying observations get small weight.

The forward search combines robust and efficient estimators, and it is based on three steps:

(1) use of very robust methods to sort and split the data into a clean outlier-free small subset, generally called "clean data set" (CDS), and a bigger subset of potential outliers;
(2) definition of a criteria by which new observations are introduced into the CDS until all observations are included;
(3) monitor and usage of several statistics of interest during the whole search.

Unlike the method of Van Dijk et al. (1999), that computes a robust test combining the generalized method of moments and a backward approach to outlier detection, our technique is not affected by masking and swamping effects related to multiple outliers identification. Atkinson and Riani (2000) and Atkinson et al. (2004) show that the forward search helps to reveal the structure of the data. Therefore, with the forward search it is possible to identify multiple outliers even when an outlier masks other outliers (masking effect) and it avoids the problem of incorrect identification of genuine observations as outliers (swamping effect).