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Optimal tests against the alternative hypothesis of panel unit roots

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ABSTRACT

For panel models with random individual and time effects, locally best invariant (LBI) tests are constructed for the null hypothesis of stationarity against the alternative hypothesis of panel unit root. Finite sample properties of the proposed test and an existing test are compared by a Monte Carlo simulation.

1. Introduction

Recently, testing for panel unit roots has attracted much attention, yielding such results as Chang (2002, 2004), Im et al. (2003), Phillips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004), Shin and Kang (2006) and many others. All the above results are developed for testing the null hypothesis of unit roots. Acceptance of the null hypothesis does not necessarily imply the presence of unit roots, for which tests against the alternative hypothesis of unit roots would be more suitable.

Following the spirit of Kwiatkowski et al. (1992) and Saikkonen and Luukkonen (1993) for testing against time series unit root, Hadri (2000) developed a locally best invariant (LBI) test against panel unit root for a model in which error terms are cross-sectionally uncorrelated. The test of Hadri (2000) would be invalid for models with cross-sectionally correlated errors, which are very common in the real world as indicated by the studies of Chang (2002, 2004), Phillips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004), and Shin and Kang (2006). For testing the null hypothesis of unit root, optimality was addressed by Ploberger and Phillips (2002) and Moon et al. (2007).

Here, we construct LBI tests for panel models consisting of individual effects, time effects, and possibly I(1) panel components. The models allow common cross-sectional correlation via time effects. As indicated in Hsiao (1986) and many other books, panel models with random individual and time effects are so widely used that it would be important to develop optimal tests for these models. LBI tests are constructed for testing zero of the variance of the possibly I(1) components. A Monte Carlo experiment reveals that, for models with cross-sectionally correlated errors, the proposed test has more stable sizes and better powers than the test of Hadri (2000).

2. LBI tests

We first consider a panel model

$$\begin{aligned} y_{it} &= \alpha_i + \beta_t + x_{it} + e_{it}, \\ x_{it} &= x_{i,t-1} + u_{it}, \end{aligned} \tag{1}$$

$t = 1, \ldots, T, i = 1, \ldots, n$, where $\{y_{it}, t = 1, \ldots, T, i = 1, \ldots, n\}$ is the set of observations on n panel units over time interval $t = 1, \ldots, T$; α_i, β_t, e_{it}, u_{it} are independent identically distributed (*i.i.d.*) zero mean errors having variances σ_α^2,

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$\sigma_\beta^2, \sigma_e^2, \sigma_u^2$, respectively; and $x_{i0} = 0$. At the end of this section extensions to the case of serially correlated u_{it} and e_{it} are discussed. Note that α_i and β_t represent individual effect and time effect respectively. Due to β_t, the error term $\alpha_i + \beta_t + e_{it}$ has a common cross-sectional correlation $\sigma_\beta^2/(\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_e^2)$. In the later part of this section, extensions to other models with time trend or factor analytic errors are made.

We are interested in testing the null hypothesis $H_0 : \sigma_u^2 = 0$ against $H_1 : \sigma_u^2 > 0$. Note that under H_0, $x_{it} = 0$ for each i, t and hence $y_{it} = \alpha_i + \beta_t + e_{it}$ has no unit root. On the other hand, under H_1, $x_{it} = \sum_{s=1}^{t} u_{is}$ with $\mathrm{var}(u_{it}) > 0$, and hence $y_{it} = \alpha_i + \beta_t + x_{it} + e_{it}$ has unit root for each i. We develop an LBI test. A related work is that of Hadri (2000) who considered a model without time effect β_t. Therefore, in the model of Hadri (2000), the error terms are cross-sectionally uncorrelated.

Letting $y_t = (y_{1t}, \ldots, y_{nt})'$, $x_t = (x_{1t}, \ldots, x_{nt})'$, $e_t = (e_{1t}, \ldots, e_{nt})'$, $Y = (y_1', \ldots, y_T')'$, $X = (x_1', \ldots, x_T')'$, $e = (e_1', \ldots, e_T')'$, $\alpha = (\alpha_1, \ldots, \alpha_n)'$, $\beta = (\beta_1, \ldots, \beta_T)'$, we have

$$Y = 1_T \otimes \alpha + \beta \otimes 1_n + X + e,$$

where $1_k = (1, \ldots, 1)'$ is a $k \times 1$ vector of ones and $\otimes$ denotes the Kronecker product. Since $\mathrm{var}(1_T \otimes \alpha) = \sigma_\alpha^2(J_T \otimes I_n)$, $\mathrm{var}(\beta \otimes 1_n) = \sigma_\beta^2(I_T \otimes J_n)$, $\mathrm{var}(X) = \sigma_u^2(\Gamma \otimes I_n)$, $\mathrm{var}(e) = \sigma_e^2 I_{nT}$, we have

$$\mathrm{var}(Y) = \sigma_\alpha^2(J_T \otimes I_n) + \sigma_\beta^2(I_T \otimes J_n) + \sigma_u^2(\Gamma \otimes I_n) + \sigma_e^2 I_{nT},$$

where $\Gamma = [\min(t, s)]$ is the $T \times T$ matrix with (t, s) element $\min(t, s)$ and $J_k = 1_k 1_k'$ is the $k \times k$ matrix of ones.

King (1980), King and Hiller (1985), Nabeya and Tanaka (1988) and others provided sufficient conditions for local optimality of a test rejecting H_0 for large derivative of log density of a maximal invariant. The conditions are, if translated to model (1), (i) after eliminating nuisance parameter effects by considering a suitable transformation of Y using orthogonal projection, the transformed random vector has mean 0 and covariance matrix of the form $\sigma_e^2[I + A(\sigma_u^2)]$ with $\partial A(\sigma_u^2)/\partial\sigma_u^2|_{\sigma_u^2=0}$ being a known matrix A_0; (ii) the distribution of the transformed vector does not depend on nuisance parameters $\sigma_\alpha^2, \sigma_\beta^2$.

We eliminate the nuisance parameters σ_α^2 and σ_β^2 in $\mathrm{var}(Y)$ by a projection $\tilde{Y} = PY$ given by

$$P = I_{Tn} - \frac{1}{T}(J_T \otimes I_n) - \frac{1}{n}(I_T \otimes J_n) + \frac{1}{Tn}(J_T \otimes J_n)$$

leading to the following transformed observation $\tilde{y}_{it}$:

$$\tilde{y}_{it} = y_{it} - \bar{y}_{i.} - \bar{y}_{.t} + \bar{y},$$

where $\tilde{Y} = (\tilde{y}_{11}, \ldots, \tilde{y}_{nT})'$, $\bar{y}_{i.} = T^{-1}\sum_{t=1}^{T} y_{it}$, $\bar{y}_{.t} = n^{-1}\sum_{i=1}^{n} y_{it}$, and $\bar{y} = (Tn)^{-1}\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$. Since $PP = P$ with rank $(nT - T - n + 1)$, we can choose a $Tn \times (Tn - T - n + 1)$ matrix H such that $P = HH'$ and $H'H = I_{Tn-T-n+1}$ with

$$\mathrm{var}(H'Y) = \sigma_e^2\left[I_{Tn-T-n+1} + \lambda H'(\Gamma \otimes I_n)H\right],$$

where $\lambda = \sigma_u^2/\sigma_e^2$. The covariance matrix of $H'Y$ is free from the nuisance parameters $\sigma_\alpha^2, \sigma_\beta^2$. Therefore, conditions (i) and (ii) of local optimality are satisfied, enabling us to construct an LBI test.

An LBI test is given by rejecting H_0 for large derivative of log density of a maximal invariant, whose details are described in more detail. The hypothesis $H_0 : \lambda = 0$ is invariant under the group G of transformation $Y \to aY$ where a is a positive real number. Let $W = H'Y$. Then the maximal invariant for the group G is given by $S(W) = W/\|W\|$. We consider the class of elliptically symmetric distributions for $H'Y$ which contains normal distributions as special cases. The density of $S(W)$ is given by

$$f(S(W)|\lambda) = c(n, T)|I_N + \lambda H'(\Gamma \otimes I_n)H|^{-1/2}\left\{\frac{W'(I_N + \lambda[H'(\Gamma \otimes I_n)H])^{-1}W}{W'W}\right\}^{-N/2},$$

where $|A|$ denotes the determinant of matrix A, $c(n, T)$ is a constant depending on the sample size and $N = Tn - T - n + 1$ [see Kariya (1980)]. According to Ferguson (1967, p. 235) an LBI test rejects H_0 if $\partial \ln f(S(W)|\lambda)/\partial\lambda$ is large, which is equivalent to ($\frac{Y'P'(\Gamma\otimes I_n)PY}{Y'PY} >$ constant) or

$$S = T^{-2}\sum_{i=1}^{n}\sum_{t=1}^{T}\left\{\sum_{s=1}^{t}\tilde{y}_{is}\right\}^2/\hat{\sigma}_e^2 > \text{constant},$$

where $\hat{\sigma}_e^2 = \sum_{i=1}^{n}\sum_{t=1}^{T}\tilde{y}_{it}^2/(Tn - n - T + 1)$.

The null distribution of the proposed LBI test does not depend on nuisance parameters $\sigma_\alpha^2, \sigma_\beta^2, \sigma_e^2$ both in finite sample and in large sample. The null distribution of S does not depend on $\sigma_\alpha^2, \sigma_\beta^2$ because S remains the same regardless of α_i and β_t. Therefore, the null distribution of S is the same as S constructed from the model with $\alpha_i = \beta_t = 0$, i.e., $y_{it} = e_{it}$. Since S is scale invariant, its distribution is free from σ_e^2. It only depends on sample sizes n and T so that the proposed LBI test can be considered as uniform in sense that its optimality holds uniformly in the nuisance parameters for common cross-sectional correlation. Also, according to Kariya (1980), the null distribution of the LBI test under elliptically symmetric assumption for the marginal distribution of $H'Y$ is the same as that under the normal distribution. The limiting null distribution of S is given in the following Lemma. The proof of this lemma is straightforward and is omitted.