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Leverage, heavy-tails and correlated jumps in stochastic volatility models[☆]

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ABSTRACT

Efficient and fast Markov chain Monte Carlo estimation methods for the stochastic volatility model with leverage effects, heavy-tailed errors and jump components, and for the stochastic volatility model with correlated jumps are proposed. The methods are illustrated using simulated data and are applied to analyze daily stock returns data on S&P500 index and TOPIX. Model comparisons are conducted based on the marginal likelihood for various SV models including the superposition model.

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1. Introduction

The stochastic volatility (SV) models have been widely used to model a changing variance of time series in financial econometrics (e.g., Ghysels et al. (2002) and Raggi (2005)). Various generalizations of the standard SV model have emerged and their model-fittings have been investigated especially in high-frequency financial data. Among such generalizations, the leverage effect, jump components and heavy-tailed errors in asset returns are well-known to be important in the recent literature (Chib et al., 2002; Jacquier et al., 2004; Yu, 2005; Omori et al., 2007; Berg et al., 2004). It has been pointed out in many empirical studies that asset returns data have heavier tails than those of normal distributions. The SV model with Student-*t* errors (SVt) is one of the most popular basic models to account for heavier tailed returns. However, it has been found insufficient to express the tail fatness of returns, and the jump components, which may be correlated, have recently been introduced to explain the tail behavior (Eraker et al., 2003). The jump component is considered to be a discretization of a Lévy process which is also widely used in the continuous time modelling of financial asset pricing. Eraker (2004) showed the empirical performance of jump diffusion models of stock price dynamics and applied them to options and returns data. Chernov et al. (2003); Raggi and Bordignon (2006) compares various different specifications of jump diffusions in the SV model in their empirical studies. We also refer to another remarkable jump specification in the GARCH model, discussed by Chan and Maheu (2002); Maheu and McCurdy (2004), which incorporates the autoregressive conditional jump intensity parameterization.

Focusing the estimation method, Kim et al. (1998) develops a fast and reliable Markov chain Monte Carlo (MCMC) algorithm for the SV model. Their impressive method, called *mixture sampler*, has been widely used in the SV literature and extended in various ways. In the context of the extension of their method, Chib et al. (2002) estimates the SV model with jumps and Student-*t* errors (SVJt) (but without leverage effect). The leverage effect refers to the increase in volatility following a previous drop in stock returns, and modelled by the negative correlation coefficient between error terms of stock returns and the volatility (e.g. Black (1976), Nelson (1991), Bartolucci and DeLuca (2003), Yu (2005), Omori et al. (2007) and

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Omori and Watanabe (2008)). The SV model with leverage effect (SVL) is also called the asymmetric stochastic volatility model. Omori et al. (2007) constructs the efficient mixture sampler for the SV model with leverage effect and Student-t errors (SVLt) (but without jumps) and demonstrates some empirical results.

In the line of developing the mixture sampler of Kim et al. (1998) for more suitable models to detect the complicated empirical structure in financial market, this paper discusses the SV models with leverage, jump components and heavy-tailed errors (SVLJt) jointly.

We consider the SV model given by

$$y_t = k_t\gamma_t + \sqrt{\lambda_t}\varepsilon_t \exp(h_t/2), \quad t = 1, \dots, n, \tag{1}$$
$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t, \quad t = 1, \dots, n-1, \tag{2}$$

where y_t is a response, h_t is an unobserved log-volatility, $|\phi| < 1$, $h_1 \sim N(0, \sigma^2/(1-\phi^2))$,

$$\begin{pmatrix}\varepsilon_t \\ \eta_t\end{pmatrix} \sim N(0, \Sigma), \quad \text{and} \quad \Sigma = \begin{pmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{pmatrix}.$$

The leverage effect measured by the correlation coefficient ρ is expected to be negative as reported in several empirical studies (Yu, 2005; Omori et al., 2007). The correlation coefficient $\rho = 0$ implies the SV model without leverage effect. The $k_t\gamma_t$ represents a jump component in the measurement Eq. (1). The γ_t is a jump flag defined as a Bernoulli random variable such that

$$\pi(\gamma_t = 1) = \kappa, \quad \pi(\gamma_t = 0) = 1 - \kappa, \quad 0 < \kappa < 1,$$

and the k_t is a jump size specified by

$$\psi_t \equiv \log(1 + k_t) \sim N(-0.5\delta^2, \delta^2), \tag{3}$$

following Andersen et al. (2002); Chib et al. (2002) where the jump parameter κ and δ are unknown and to be estimated. We denote the SV and SVL models with jumps as the SVJ and SVLJ models respectively.

The measurement error $\sqrt{\lambda_t}\varepsilon_t$ is assumed to follow the heavy-tailed Student-t distribution with unknown degrees of freedom ν by letting

$$\lambda_t^{-1} \sim \text{Gamma}(\nu/2, \nu/2). \tag{4}$$

We may also assume $\log\lambda_t \sim N(-0.5\tau^2, \tau^2)$ to obtain the lognormal scale mixture as in Omori et al. (2007), but we illustrate our algorithm using the Gamma scale mixture given by (4). When $\lambda_t \equiv 1$ for all t, the model reduces to the SV or SVL model with normal errors.

The contribution of this paper comprises two parts. First, we develop the efficient and fast MCMC parameter estimation method for the SVLJt model (SV model with leverage, jumps and Student-t errors) extending Chib et al. (2002) and Omori et al. (2007). Second, we extend it to the SV model with correlated jumps, which have recently been popular in financial literature.

We illustrate our approach using simulated data and apply it to the stock returns data of S&P500 index and TOPIX (Tokyo Stock Price index). Using the Bayesian approach of marginal likelihood computation, we compare various candidate models over the class of SV model with jumps, leverage and heavy-tails. The superposition model is also considered.

The rest of paper is organized as follows. In Section 2 we discuss the MCMC estimation for our SV model with jumps, leverage and heavy-tails. Section 3 illustrates our method using simulated data. In Section 4, we extend it to the SV model with correlated jumps. In Section 5, we apply our proposed method to the daily asset returns data of S&P500 and TOPIX. Section 6 concludes the paper.

2. SV model with jumps, leverage and heavy-tails

The well-known difficulty of estimating the discrete-time SV model is that the likelihood function is not easily available. It is possible to compute the likelihood using a simulation-based method for a given set of parameters, which is called a particle filter. But it requires a computational burden since we need to repeat the particle filter many times to evaluate the likelihood function for each set of parameters until we reach the maximum. To overcome this difficulty, we take the Bayesian estimation approach and propose the MCMC methods (e.g., Chib and Greenberg (1996)) for a precise and efficient estimation of the SVLJt model.

2.1. Auxiliary mixture sampler

Following Omori et al. (2007), we define $y_t^* = \log(y_t - k_t\gamma_t)^2 - \log\lambda_t$, $d_t = \text{sign}(y_t - k_t\gamma_t) = I(\varepsilon_t > 0) - I(\varepsilon_t \le 0)$, which rewrites Eq. (1) as

$$y_t^* = h_t + \xi_t, \tag{5}$$

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