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A frequency domain test for detecting nonstationary time series

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HIGHLIGHTS

- We propose a frequency domain test against nonstationarity in time series.
- The test is based on comparing the goodness of fit in log-periodogram regression.
- The regression model is the varying coefficient fractionally exponential model.
- The test is applicable for dynamic changes of both short and long range dependences.
- The finite sample test distribution is approximated by bootstrap.

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ABSTRACT

We propose a frequency domain generalized likelihood ratio test for testing nonstationarity in time series. The test is constructed in the frequency domain by comparing the goodness of fit in the log-periodogram regression under the varying coefficient fractionally exponential models. Under such a locally stationary specification, the proposed test is capable of detecting dynamic changes of short-range and long-range dependences in a regression framework. The asymptotic distribution of the proposed test statistic is known under the null stationarity hypothesis, and its finite sample distribution can be approximated by bootstrap. Numerical results show that the proposed test has good power against a wide range of locally stationary alternatives.

1. Introduction

The stationarity assumption plays an essential role in making inference for time series analysis. Under this assumption, statistical properties such as consistency and asymptotic normality hold for standard inference in time series analysis, based only on one realization. However, the stationarity assumption might not be realistic in practice, especially when the observed time span is long or the series is apparently affected by some natural factors. For example, the influenza-like illness incidence has different dynamics during the epidemic period and non-epidemic period; the tree ring data exhibit time varying dynamics due to different weather and environmental conditions over the years. As another example in finance, volatilities of returns usually exhibit strong dependencies but the level of persistence is likely to change over time due to numerous unpredictable economic factors (Ray and Tsay, 2002).

In order to validate the analysis based on stationary modeling, some preliminary checking of stationarity is necessary. Usually, visualizing the time series plot is a useful way of spotting certain types of nonstationarity such as the mean and

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variance change over time, but this naive method makes somehow difficult to explore more complex dynamic changes in the dependence structures. An alternative approach for checking stationarity is through the hypothesis testing. In the literature, there are various tests under different specifications of nonstationarity. Early development focused on the change point problem subject to a specific parameter under a fully parametric setting (Inclán and Tiao, 1994). Recent research focuses on nonstationary modeling with slowly varying dependence structures. The pioneering work in this area was given by Priestley (1965) by introducing the evolutionary spectrum with the power-frequency analog of stationary spectrum under a time-dependent framework. Later, Priestley and Subba Rao (1969) further developed an analysis of variance (ANOVA) procedure to test the homogeneity of evolutionary spectra. This test is easy to implement and is particularly powerful for detecting nonstationarity against a uniformly modulated process. However, due to completely unspecified time-frequency interactions in the ANOVA model, this ANOVA test is less powerful against other types of nonstationarity, such as the class of varying coefficient AR models.

In contrast to Priestley's approach, Dahlhaus (1997) gave an alternative specification for the locally stationary process with smoothly varying spectra. The well-developed asymptotic theories under this local stationarity framework bring about its popularity among other approaches in the literature (e.g. Adak, 1998; Guo et al., 2003; Nason et al., 2000; Davis et al., 2006). Accordingly, several testing procedures for detecting nonstationarity are constructed under Dahlhaus's locally stationary framework. Sakiyama and Taniguchi (2003) proposed the likelihood ratio test, Wald test and Lagrange multiplier test for testing a specific structure in a locally stationary Gaussian process. These likelihood-based tests require fully specified hypotheses and the results are often sensitive to model mis-specification. Sergides and Paparoditis (2009) proposed a more general procedure for detecting local stationarity based on a goodness of fit measure, which allows a semiparametric specification for the null model against a unspecified locally stationary alternative model. Similar idea was also applied for validating and monitoring stationarity for a process over time (Paparoditis, 2010).

Among the studies for testing nonstationarity, most of the interest focuses on the changes of short-range dependence while the issue concerning long-memory nonstationarity is rarely addressed. Regarding such issue, change point approaches have been suggested under fully parametric settings (e.g. Beran and Terrin, 1996; Ray and Tsay, 2002). Recently, Beran (2009) incorporated long-memory structure into the locally stationary process framework in a time domain approach, which mainly concerned on estimation problems rather than hypothesis testing issues.

In this paper, we propose a general testing procedure which aims to detect the dynamic change for both short-memory and long-memory structures. Our approach is built on a semiparametric class of models, called varying-coefficient fractionally exponential (VC-FEXP) models, whose specification is extended from a stationary fractionally exponential (FEXP) model of Beran (1993) by allowing parameters in the spectra to vary smoothly over time. This VC-FEXP model is appealing not only because the specification is very flexible but also because the corresponding inference can be carried out in a regression framework. The regression used here is a time-varying version of the log-periodogram regression. Under this regression framework, we suggest a generalized goodness-of-fit test to detect various aspects of nonstationarity.

The rest of the paper is organized as follows. In Section 2, the proposed VC-FEXP model is introduced, under which the varying-coefficient log-periodogram regression and the corresponding local linear fitting are described. Section 3 defines the hypotheses and the test statistic for detecting nonstationarity. Numerical results are reported in Section 4, including a real data example. Section 5 is the summary.

2. Varying coefficient FEXP models

For the purpose of testing stationarity, we first suggest a general family of nonstationary models represented via time-varying spectra in a locally stationary process framework. This general alternative model, called the varying-coefficient FEXP (VC-FEXP), is a natural generalization of the FEXP model (Beran, 1993) by allowing parameters to vary smoothly over time. Specifically, a VC-FEXP model is defined via the following time-varying spectra:

$$f_t(\omega) \equiv f(\omega; \boldsymbol{\theta}(u)) = |1 - e^{i\omega}|^{-2d(u)} \exp\left\{\sum_{k=0}^{p} \theta_k(u)\psi_k(\omega)\right\}, \quad u = t/T \in [0, 1], \tag{1}$$

where the time index t is represented by the re-scaled time $u = t/T$, $\psi_0(\omega) = 1$ and $\{\psi_k(\cdot) : k = 1, \ldots, p\}$ are some continuous and even functions defined on $[-\pi, \pi]$ satisfying $\int_0^\pi \psi_k^2(\omega)d\omega < \infty$. $d(u)$ and $\boldsymbol{\theta}(u) \equiv (\theta_0(u), \ldots, \theta_p(u))'$, having continuous second derivatives, characterize the long-range and short-range dependence structures of the underlying process respectively.

When $d(u) = d, \theta_k(u) = \theta_k$ and $\psi_k(\omega) = \cos(k\omega)$, this generalized model (1) reduces to the FEXP model (Beran, 1993) as a special case, which has a very flexible form of approximating a variety of short-range and long-range dependence structures for a stationary process. In addition, FEXP plays an important role on long-memory parameter estimation (Beran, 1993; Hurvich and Brodsky, 2001). Inherited from these advantages associated with FEXP models, the VC-FEXP model not only provides a flexible semiparametric family for approximating locally stationary processes but also serves as a general alternative model for testing nonstationarity by examining the homogeneity of the coefficient functions $d(u)$ and $\boldsymbol{\theta}(u)$.

Regarding the model fitting, since the VC-FEXP model is specified in the frequency domain, it is natural to use the log-periodogram regression for estimating the time-varying spectra. In Section 2.1, a customized log-periodogram regression