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Bayesian analysis of two-piece location–scale models under reference priors with partial information

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ABSTRACT

Bayesian estimators are developed and compared with the maximum likelihood estimators for the two-piece location–scale models, which contain several well-known distributions such as the asymmetric Laplace distribution, the two-piece normal distribution, and the two-piece Student-t distribution. For the validity of Bayesian analysis, it is essential to use priors that could lead to proper posterior distributions. Specifically, reference priors with partial information have been considered. A sufficient and necessary condition is established to guarantee the propriety of the posterior distribution under a general class of priors. The performance of the proposed approach is illustrated through extensive simulation studies and real data analysis.

1. Introduction

The use of skewed distributions is an attractive option for modelling data when the assumption of symmetry is not appropriate; see, for example, Azzalini and Capitanio (1999), Azzalini and Capitanio (2014) and Jones (2015), among others. As an illustration, it is widely known that the asymmetric Laplace distribution (ALD), a special case of this family, has received much attention in a wide range of disciplines, such as economics (Zhao et al., 2007), engineering (Kotz et al., 2001), financial analysis (Kozubowski and Podgórski, 2001), medical study (Purdom et al., 2005), and microbiology (Rubio and Steel, 2011). In recent years, numerous techniques have been developed to derive new skewed distributions, mainly based on a modification of various symmetric distributions. We do not review them here in detail but point the interested readers to Azzalini (1985), Fernández and Steel (1998) and Nadarajah and Kotz (2003), to name just a few.

Due to their simplicity and fitting real data quite well in practice, the two-piece location–scale models have been paid considerable attention in the literature. Besides the ALD, many other sub-distributions of the two-piece location–scale models have also been discussed, such as the skewed Student-t distribution which has been discussed by Fernández and Steel (1998), the ϵ-skew normal distribution which has been studied by Mudholkar and Hutson (2000). In the absence of prior knowledge, an objective prior such as the Jeffreys prior is often preferred to conduct Bayesian inference. Recently, Rubio and Steel (2014) derived the Jeffreys and independence Jeffreys priors for several families of the skewed distributions. Unfortunately, it has been shown that these Jeffreys priors result in improper posterior distribution for some sub-distributions, such as the skewed Student-t distribution. Of particular note is that from Rubio and Steel (2014), several discussants advocated the use of reference priors proposed by Berger and Bernardo (1992), which are very difficult to calculate for the two-piece location–scale models. However, reference priors with partial information (for short, RPPI)

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firstly proposed by Sun and Berger (1998) share the same idea as reference priors, and are easier to derive. Therefore, we are interested in deriving RPPI for the family of two-piece distributions and obtaining conditions for the propriety of the corresponding posterior distribution.

The use of RPPI is quite attractive in applied situations because we usually have some partial prior information for several parameters. Thus, we just need to find a conditional prior for the remaining unknown parameters based on available information. For instance, Berger et al. (2001) showed that for the range parameter in the spatial model, the frequentist coverage probability of the credible intervals based on the RPPI is better than the one in terms of the Jeffreys prior. Ferreira and Suchard (2008) illustrated that for elapsed times in continuous-time Markov chains, the frequentist coverage of the credible intervals of the parameters based on the RPPI are better than the ones from other priors. Demortier et al. (2010) discussed Bayesian inference for the high energy physics problems by applying the RPPI for both single-count and multiple-count models and obtained a nice frequentist coverage probability. In this paper, we derive RPPI for the two-piece location–scale models and show that some of them lead to proper posterior distributions. In particular, a sufficient and necessary condition for the propriety of the posterior distribution is provided under a general class of priors.

The rest of this paper is organized as follows. In Section 2, we describe the two-piece location–scale models and present several skewed distributions from different parameterizations. In Section 3, we derive several RPPI for these distributions and study the propriety of the posterior distributions for the ALD in detail. In Section 4, the performance of our approach is illustrated through extensive simulation studies and a real data application. Finally, some concluding remarks are provided in Section 5, with proofs given in Appendix A.

2. Two-piece location–scale models

The framework of the two-piece location–scale models was established by Rubio and Steel (2014). Let $f(y \mid \mu, \sigma)$ be a symmetric and absolutely continuous density with support on $\mathbb{R}$, location parameter $\mu \in \mathbb{R}$, and scale parameter $\sigma \in \mathbb{R}^+$. The probability density function (pdf) of these models as proposed by Rubio and Steel (2014) has the following form:

$$g(y \mid \mu, \sigma, \gamma) = \frac{2}{\sigma\left[a(\gamma) + b(\gamma)\right]} \left\{ f\left(\frac{y-\mu}{\sigma b(\gamma)}\right) I_{(-\infty,\mu)}(y) + f\left(\frac{y-\mu}{\sigma a(\gamma)}\right) I_{[\mu,\infty)}(y) \right\}, \tag{1}$$

where $\gamma \in \Gamma$ is an asymmetry parameter with the set Γ depending on the choice of $\{a(\cdot), b(\cdot)\}$, $a(\cdot)$ and $b(\cdot)$ are known and positive functions, and both are differentiable such that

$$0 < |\lambda(\gamma)| < \infty, \quad \text{with} \quad \lambda(\gamma) = \frac{d}{d\gamma} \log\left[\frac{a(\gamma)}{b(\gamma)}\right].$$

The density in (1) was also presented by Arellano-Valle et al. (2005) as a general class of asymmetric distributions, including the entire family of univariate symmetric unimodal distributions as a special case. In this paper, we mainly focus on the case in which $f(\cdot)$ belongs to the class of scale mixture of normals, which includes three common models in terms of $\{a(\gamma),\ b(\gamma)\}$: the inverse scale factors model with $\{a(\gamma) = \gamma,\ b(\gamma) = 1/\gamma\}$ (Fernández and Steel, 1998), the ϵ-skew model with $\{a(\gamma) = 1-\gamma,\ b(\gamma) = 1+\gamma\}$ (Mudholkar and Hutson, 2000), and the ALD with $f(\cdot)$ being the standard Laplace distribution, and $\{a(\gamma) = 1/\gamma,\ b(\gamma) = 1/(1-\gamma)\}$ (Yu and Zhang, 2005).

Mudholkar and Hutson (2000) discussed a particular case of the ϵ-skew model, the so-called ϵ-skew-normal distribution, and they considered Bayesian analysis by adopting a subjective prior for μ, with fixed σ and γ. Yu and Moyeed (2001) considered Bayesian quantile regression by employing a likelihood function which is based on the ALD. From a practical point of view, a prior with some objective information is more reasonable due to the lack of prior information in various applications. These observations motivate us to consider alternative priors with objective information for all model parameters.

3. Reference priors with partial information

Due to the lack of prior knowledge about the unknown parameters, we often have a preference for the use of objective priors. One of the most widely used noninformative priors is the Jeffreys prior, which is proportional to the square root of the determinant of the Fisher information matrix of the model. The Jeffreys prior enjoys the invariant property under any one-to-one reparameterization of the model. For notational simplicity, we use the same notations as Rubio and Steel (2014). Define

$$\alpha_1 = \int_0^\infty \left[\frac{f'(t)}{f(t)}\right]^2 f(t)\, dt,$$

$$\alpha_2 = 2\int_0^\infty \left[1 + t\frac{f'(t)}{f(t)}\right]^2 f(t)\, dt,$$

$$\alpha_3 = \int_0^\infty t\left[\frac{f'(t)}{f(t)}\right]^2 f(t)\, dt.$$