Computational Statistics and Data Analysis 56 (2012) 1566–1580

Contents lists available at SciVerse ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Nonparametric estimation of non-stationary velocity fields from 3D particle tracking velocimetry data

Michael Kohler[a], Adam Krzyżak[b,*]

[a] Fachbereich Mathematik, Technische Universität Darmstadt, Schlossgartenstr. 7, 64289 Darmstadt, Germany

[b] Department of Computer Science and Software Engineering, Concordia University, 1455 De Maisonneuve Blvd. West, Montreal, Quebec, Canada H3G 1M8

ARTICLE INFO

Article history:
Received 16 May 2011
Received in revised form 25 September 2011
Accepted 27 September 2011
Available online 8 October 2011

Keywords:
Consistency
Nonparametric regression
Particle tracking velocimetry

ABSTRACT

Nonparametric estimation of nonstationary velocity fields from 3D particle tracking velocimetry data is considered. The velocities of tracer particles are computed from their positions measured experimentally with random errors by high-speed cameras observing turbulent flows in fluids. Thus captured discrete data is plugged into a smoothing spline estimate which is used to estimate the velocity field at arbitrary points. The estimate is further smoothed over several time frames using the fixed design kernel regression estimate. Consistency of the resulting estimate is investigated. Its performance is validated on the real data obtained by measuring a fluid flow of a liquid in a (rotating) square tank agitated by an oscillating grid.

© 2011 Elsevier B.V. All rights reserved.

1. Introduction

In recent years there has been tremendous progress in accurate and fast measurement techniques in fluid mechanics. This resulted in a large amount of data; processing and interpretation of this data presently require the development of new statistical tools.

In this article, we analyze data produced by the 3D Particle Tracking Velocimetry (3D-PTV); see, e.g., Raffel et al. (1998). This technique allows visualization of a flow by recording the laser light scattered by naturally buoyant tracer particles in a fluid and subsequently using it to determine positions of the particles in consecutive frames. To do this 3D-PTV fits short trajectories of the particles to the observed pictures. These trajectories consist of approximately 20 time steps and are modeled by cubic splines. From these trajectories, the estimates of the position and the velocity of the tracer particles are derived (cf., e.g., Lüthi et al., 2005). Thus data is produced which contains positions of particles and corresponding values of the fluid velocity field at these positions. This data contains two kinds of errors: first, errors due to measurement errors for the locations of the tracer particles, and second, errors due to fitting of the trajectories to these locations of the tracer particles. In this paper, we want to use this data to estimate the velocity field at arbitrary locations and times.

Experimental studies on estimation of velocity fields in turbulent flows have been carried out, among others, by Guala et al. (2008), Kunnen et al. (2010), Lüthi et al. (2005), Messio et al. (2008) and Speetjens et al. (2004). These researchers used kernel regression and local linear kernel regression estimates to smooth and interpolate the observed data. No theoretical analysis of the estimates was provided.

Nonparametric regression estimation has been studied over many years. Two main approaches were developed: random design approach and fixed design approach. The most popular estimates for the random design approach include kernel regression estimate Nadaraya (1964, 1970), Watson (1964), Devroye and Wagner (1980), Stone (1977, 1982) and Devroye

* Corresponding author. Tel.: +1 514 848 2424X3007; fax: +1 514 848 2830.
E-mail addresses: kohler@mathematik.tu-darmstadt.de (M. Kohler), krzyzak@cs.concordia.ca (A. Krzyżak).

0167-9473/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.csda.2011.09.025

and Krzyżak (1989), partitioning regression estimate Györfi (1981), Beirlant and Györfi (1998), nearest neighbor regression estimate Devroye (1982), Devroye et al. (1994), Mack (1981), Zhao (1987), and estimators based on orthogonal series and those using Bernstein and Bernstein–Durrmeyer polynomials Rafajłowicz (1987) and Rafajłowicz and Rafajłowicz-Skubalska (1999). The main theoretical results are summarized in the monograph Györfi et al. (2002). For the survey of fixed design regression estimates, we refer to the monograph by Eubank (1999).

In this paper, we pose the problem of recovering velocity fields at arbitrary locations and times as a non-stationary regression estimation problem with regression functions changing in time. The regression functions are estimated by smoothing spline estimates which are subsequently smoothed in time domain using the fixed design kernel regression estimate.

We prove consistency of the estimates and apply them to real data obtained from the 3D-PTV measuring a time-dependent velocity field in a (rotating) water tank agitated by an oscillating grid.

2. Definition of the estimates

Let $(X_t, Y_t)(t \in [0, 1])$ be $\mathbb{R}^d \times \mathbb{R}^d$-valued random vectors defined on the same probability space. Let the corresponding time dependent d-dimensional velocity field

$$m : [0, 1] \times \mathbb{R}^d \to \mathbb{R}^d$$

be given by

$$m(t, x) = \mathbf{E}\{Y_t | X_t = x\},$$

and denote the distribution of X_t by μ_t. For $N \in \mathbb{N}$ we consider equidistant time points

$$t_k = t_k(N) = \frac{k}{N} \quad (k = 0, \dots, N)$$

and we assume that for each time point t_k, we are given a velocity field sample

$$\mathcal{D}_{n_{t_k}} = \{(X_1^{(t_k)}, Y_1^{(t_k)}), \dots, (X_{n_{t_k}}^{(t_k)}, Y_{n_{t_k}}^{(t_k)})\}.$$

Let $k \in \mathbb{N}$ with $2k > d$ and denote by $W^k(\mathbb{R}^d)$ the Sobolev space containing all functions $f : \mathbb{R}^d \to \mathbb{R}^d$ where all derivatives of total order k of all components are in $L^2(\mathbb{R}^d)$. The condition $2k > d$ implies that the functions in $W^k(\mathbb{R}^d)$ are continuous and hence the evaluation of a function at a point is well defined. Let $m_{n_{t_k}}^{(t_k)}(\cdot) = m_{n_{t_k}}^{(t_k)}(\cdot, \mathcal{D}_{n_{t_k}})$ be the smoothing spline estimate of $m(t_k, \cdot)$ defined by

$$\tilde{m}_{n_{t_k}}^{(t_k)}(\cdot) = \arg \min_{f \in W^k(\mathbb{R}^d)} \left[\frac{1}{n_{t_k}} \sum_{i=1}^{n_{t_k}} \| Y_i^{(t_k)} - f(X_i^{(t_k)}) \|_2^2 + \lambda_{t_k} \cdot J_k^2(f) \right] \tag{1}$$

where

$$J_k^2(f) = \sum_{\alpha_1, \dots, \alpha_d \in \mathbb{N}, \, \alpha_1 + \dots + \alpha_d = k} \frac{k!}{\alpha_1! \cdot \dots \cdot \alpha_d!} \int_{\mathbb{R}^d} \left\| \frac{\partial^k f}{\partial x_1^{\alpha_1} \dots \partial x_d^{\alpha_d}}(x) \right\|_2^2 dx, \tag{2}$$

and by

$$m_{n_{t_k}}^{(t_k)}(x) = T_{\beta_N} \tilde{m}_{n_{t_k}}^{(t_k)}(x) := \max(\min(\tilde{m}_{n_{t_k}}^{(t_k)}(x), \beta_N), -\beta_N) \quad (x \in \mathbb{R}^d). \tag{3}$$

Here $\| \cdot \|_2$ denotes the Euclidean norm in $\mathbb{R}^d$ and the truncation level $\beta_N > 0$ is a parameter of the estimate which we will choose later such that $\beta_N \to \infty (N \to \infty)$.

Let $l = \binom{d+k-1}{d}$ and let $\phi_1, \dots, \phi_l$ be all monomials $x_1^{\alpha_1} \cdot \dots \cdot x_d^{\alpha_d}$ of total degree $\alpha_1 + \dots + \alpha_d$ less than k. Define $R : \mathbb{R}_+ \to \mathbb{R}$ by

$$R(u) = \begin{cases} u^{2k-d} \cdot \log(u) & \text{if } 2k - d \text{ is even,} \\ u^{2k-d} & \text{if } 2k - d \text{ is odd,} \end{cases}$$

where $\log(z)$ is the natural logarithm of $z > 0$. It follows from Section V in Duchon (1976) that there exists a function of the form

$$\tilde{m}_{n_{t_k}}^{(t_k)}(x) = \sum_{i=1}^{n} a_i R(\|x - X_i^{(t_k)}\|_2) + \sum_{j=1}^{l} b_j \phi_j(x) \tag{4}$$

which achieves the minimum in (1), and that the coefficients $a_1, \dots, a_n, b_1, \dots, b_l \in \mathbb{R}^d$ of this function can be computed by solving d linear systems of equations. Under some additional assumptions on the $X_1^{(t_k)}, \dots, X_n^{(t_k)}$ this is also shown in Section 2.4 of Wahba (1990).

Download English Version:

https://daneshyari.com/en/article/416447

Download Persian Version:

https://daneshyari.com/article/416447

Daneshyari.com