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Progressively first-failure censored reliability sampling plans with cost constraint

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ABSTRACT

In this article, reliability sampling plans are developed for the Weibull distribution when the life test is progressively first-failure censored. We use the maximum likelihood method to obtain the point estimators of the model parameters. We propose an approach to establish reliability sampling plans which minimize three different objective functions under the constraint of total cost of experiment and given consumer's and producer's risks. The results are tabulated for selected specifications under progressive first-failure censoring scheme, and the sensitivity analysis is also studied. A Monte Carlo simulation is performed to study the accuracy of large-sample approximation.

1. Introduction

An important issue for engineers is how to collect the lifetime data of products in a limited experimental time and then to assess the reliability. In the past several decades, censoring is very common in reliability data analysis. It usually applies when the exact lifetimes are known for only a portion of the products and the remainder of the lifetimes has only partial information. Type I and type II are two of the most common censoring schemes. Consider a sample of size n units placed on a life test. In type I censoring, experiments are run over a fixed time period such that beyond this time no failures will be observed. Thus, the number of exact lifetimes observed is random. On the other hand, in type II censoring, experiments are terminated at the time of the m-th ($m \leq n$) failure observed so that $n - m$ partially observed failure times are known only to exceed certain value. Thus, the time of termination of the experiment is random. This two censoring schemes have been studied by numerous authors such as Mann et al. (1974), Meeker and Escobar (1998), and Lawless (2003).

Although type II censoring can shorten the test duration, the experimental time is still too long when products have high reliability. Johnson (1964) proposed a concept of grouping in which the experimenter might group the test units into several sets, each set as an assembly of test units, and then all the units are tested simultaneously until the first failures in each group are observed. Balasooriya (1995) indicated that in a situation where the lifetime of a product is quite high and test facilities are scarce but test material is relatively cheap, one can test mn units by testing m sets, each containing n units. The life test is then conducted by testing each of these sets of units separately until the occurrence of first failure in each set. Such a censoring scheme is called first-failure censoring. Wu et al. (2001) extended the results of Balasooriya (1995) and provided a limited failure censored life test for the Weibull distribution. Based on the first-failure censored data,

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Wu et al. (2003) constructed the exact confidence intervals and exact joint confidence regions for the parameters of the Gompertz distribution.

Note that a first-failure censoring scheme is terminated when the first failure in each set is observed. If an experimenter desires to remove some sets of test units before observing the first failures in these sets, the above described scheme will not be of use to the experimenter. The first-failure censoring does not allow for sets to be removed from test at the points other than the final termination point. However, this allowance will be desirable when some sets of the surviving units in the experiment that are removed early on can be used for some other tests. This leads us to the area of progressive censoring. In this topic, much work of statistical inference has been done by several authors including, for example, Yang and Tse (2005), Lin et al. (2006), Soliman (2008), Ng et al. (2009), and Chen and Lio (2010). An account on progressive censoring can be found in the book by Balakrishnan and Aggarwala (2000) or in the review article by Balakrishnan (2007).

Wu and Kuş (2009) combined the concepts of first-failure censoring and progressive censoring to develop a new life test plan called a progressive first-failure censoring scheme. In this scheme, one allows sets with no failure to be removed from the test before the end of the experiment. They derived the maximum likelihood estimators, exact and approximate confidence intervals for the parameters of the Weibull distribution.

To conduct a progressively censored life test more efficiently, one has to address the problem of determining optimal setting that produces the best estimation results. Recently, Burkschat (2008) established a simple property of a general optimality criterion that yields optimality of certain extremal schemes. Balakrishnan et al. (2008) investigated the optimal censoring schemes in the sense of maximization of the Fisher information in progressive type II censoring. Wu et al. (2008) discussed four selection criteria which enable us to obtain the optimum step-stress accelerated life test plans under type I progressive group-censoring. Pareek et al. (2009) studied five different criteria for finding the optimal progressive type II censoring schemes with competing risks. Wu and Huang (2010) obtained the optimal settings of a progressively group-censored life test by minimizing the asymptotic variance of mean life under the constraint that the total experimental cost does not exceed a pre-determined budget. Cramer and Schmiedt (2011) considered both trace and determinant of the Fisher information as an objective function for obtaining the optimal progressive censoring plans in the presence of competing risks.

Acceptance sampling is an important statistical tool in the area of quality control. It is used to make determination on accepting or rejecting a lot of product. One major classification of acceptance sampling plans is by attributes and variables. Attributes sampling makes decisions based on the number of defects within a lot so the quality characteristics are expressed on a "go, no-go" basis. Variables sampling makes decisions based on measurement values so the quality characteristics are measured on a numerical scale. Montgomery (2005) indicates that measurement data usually provide more information about the manufacturing process or the lot than do attributes data, and numerical measurements of quality characteristics are more useful than simple classification of the item as defective or nondefective. Therefore, acceptance sampling plans by variables are more efficient than by attributes.

Acceptance sampling plans by variables not only guarantee consumers against using defects but also offers the quality of products for producers. Producers have to perform a sampling plan to obtain the values of the quality characteristics of products in order to give assurance for consumers. In practice, an important quality characteristic is the lifetime of a product. Acceptance sampling plans used to determine the acceptability of a batch of products, with respect to their lifetimes, are known as reliability sampling plans. In the literature, Balasooryia and Saw (1998) and Tse and Yang (2003) derived reliability sampling plans for exponential and Weibull distributions, respectively, based on progressively type II censored data. Some other related works are, for example, Balasooriya et al. (2000), Ng et al. (2004), Chen et al. (2004), Fernández (2005), Jun et al. (2006), and Peréz-González and Fernández (2009).

In the design of reliability sampling plans with progressively first-failure censored data, one needs to decide the number of groups, the number of test units in each group, and the critical point. One practical problem arising from designing a reliability sampling plan is the cost of the experiment. However, in the literature, very few researchers considered the cost restriction when designing sampling plans. For example, Lui et al. (1993) obtained the optimal sample size for grouped exponential data. Tse et al. (2002) investigated the planning of integrated type II interval censored life tests for exponential lifetimes. Huang and Wu (2008) gave a reliability sampling plan for progressively type I interval censored life tests when the lifetime follows the exponential distribution and the cost of experiment is minimized.

The purpose of this article is to explore the optimal number of groups and the optimal number of test units in each group in conducting a reliability sampling plan with desired producer's and consumer's risks. The rest of the paper is organized as follows: Section 2 describes the formulation of a progressive first-failure censoring scheme. Section 3 derives maximum likelihood estimators (MLEs) of the parameters based on the proposed censoring scheme. Section 4 studies the design of reliability sampling plans based on the large-sample approximations. We proposes an approach to determine the number of groups and the number of test units in each group, and then sets up a reliability sampling plan with cost consideration. Section 5 applies the proposed approach to some numerical studies, sensitivity analysis, and simulations. Some conclusions and discussions are in Section 6.

2. A progressive first-failure censoring scheme

Suppose that n independent groups with k items in each group are placed on a life test with the corresponding lifetimes being identically distributed with probability density function $f(x; \theta)$ and cumulative distribution function $F(x; \theta)$, where