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Bootstrap prediction mean squared errors of unobserved states based on the Kalman filter with estimated parameters

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ABSTRACT

In the context of linear state space models with known parameters, the Kalman filter (KF) generates best linear unbiased predictions of the underlying states together with their corresponding Prediction Mean Square Errors (PMSE). However, in practice, when the filter is run with the parameters substituted by consistent estimates, the corresponding PMSE do not take into account the parameter uncertainty. Consequently, they underestimate their true counterparts. In this paper, we propose two new bootstrap procedures to obtain PMSE of the unobserved states designed to incorporate this latter uncertainty. We show that the new bootstrap procedures have better finite sample properties than bootstrap alternatives and than procedures based on the asymptotic approximation of the parameter distribution. The proposed procedures are implemented for estimating the PMSE of several key unobservable US macroeconomic variables as the output gap, the Non-accelerating Inflation Rate of Unemployment (NAIRU), the long-run investment rate and the core inflation. We show that taking into account the parameter uncertainty may change their prediction intervals and, consequently, the conclusions about the utility of the NAIRU as a macroeconomic indicator for expansions and recessions.

1. Introduction

State space models are very popular for describing the dynamic evolution of a large range of economic and financial time series in which there are unobserved variables of interest; see, for example, Fernández-Villaverde et al. (2007) who propose representing the equilibrium of an economic model using a state-space representation, Orphanides and van Norden (2002), Doménech and Gómez (2006) and Proietti et al. (2007) for estimating the unobserved output gap in several economies and Stock and Watson (2007) for a trend-cycle model with stochastic volatility fitted to US inflation, just to cite several recent empirical applications.

One of the main attractiveness of state space models is that they allow the implementation of the Kalman filter and smoothing algorithms which deliver estimates of the underlying states which, in the context of linear state space models with known parameters, are best linear unbiased. The filters also deliver the corresponding prediction mean squared errors (PMSE) which measure the uncertainty associated with the estimated states. However, in practice, the filter is run with some parameters substituted by consistent estimates. In this case, the Kalman filter PMSE do not take into account the additional uncertainty due to the parameter estimation. As a result, they underestimate the true PMSE and, consequently, the uncertainty associated with the estimates of the underlying states; see among others, Ansley and Kohn (1986), Hamilton (1986), Durbin and Koopman (2000) and Quenneville and Singh (2000).

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There are several procedures available in the literature to incorporate the parameter estimation uncertainty into the Kalman filter PMSE. First, fully Bayesian methods generate distributions of the underlying states which in a natural way incorporate the parameter uncertainty; see, for example, Carter and Kohn (1994) and Durbin and Koopman (2002). However, these procedures can be computationally complicated and time consuming in relatively large models; see Harvey (2000) and Quenneville and Singh (2000). Furthermore, they usually require particular assumptions about the conditional distribution of the parameters and states. Alternatively, as proposed by Ansley and Kohn (1986), the PMSE of the estimated underlying states can be computed by using the Delta method and the first two moments of the asymptotic distribution of the parameter estimator. There are also corrections of the Kalman filter PMSE based on the Monte Carlo integration of the distribution of the parameter estimator which is approximated by the asymptotic distribution; see Hamilton (1986). However, the asymptotic distribution can be a poor approximation to the finite sample distribution when the sample size is not large enough. Proietti et al. (2007) implements the Hamilton (1986) and Ansley and Kohn (1986) procedures to obtain standard deviations of the Euro area output gap that incorporate the parameter uncertainty. However, Quenneville and Singh (2000) show that these two procedures miss terms of the same order as their proposed corrections. Consequently, they propose enhancements of both procedures to estimate the PMSE based on second order approximations of the parameter distribution and show that the corresponding reductions of the PMSE biases are relatively small. Kass and Steffey (1989) also propose a second order approximation which is even more computationally demanding. To overcome these limitations, Quenneville and Singh (2000) propose computing the PMSE of the underlying states by using the Monte Carlo integration of the distribution of the parameter estimator where this distribution is approximated by the posterior distribution obtained in a Bayesian fashion. Although their procedure reduces the biases in the PMSE, it is proposed in the context of a particular simple model, the local level model, and can be computationally demanding for more general unobserved component models. Finally, Pfeffermann and Tiller (2005) propose using bootstrap procedures to compute PMSE in the context of the Kalman filter. Bootstrap procedures have the advantage of being computationally simple even in relatively complicated models. Furthermore, they are robust against misspecification of the error distribution; see Wall and Stoffer (2002, 2004) and Rodríguez and Ruiz (2009) for their implementation to obtain the prediction distribution of future values of the observed variables. However, the bootstrap PMSE proposed by Pfeffermann and Tiller (2005) are designed to obtain unconditional PMSE of the estimates of the underlying states. The distinction between conditional and unconditional PMSE could be important in time-varying state space models with estimated parameters; see Ansley and Kohn (1986) for arguments in favor of computing conditional PMSE.

Consequently, in this paper, we propose two new bootstrap procedures to obtain conditional PMSE of the Kalman filter estimates of the unobserved states that incorporate the parameter uncertainty. Following, Hamilton (1986) and Quenneville and Singh (2000), the new procedures are based on the Monte Carlo integration of the distribution of the parameter estimator, but instead of approximating this distribution by the asymptotic or posterior distributions, we propose to approximate it by a bootstrap distribution; see Stoffer and Wall (1991) for the bootstrap approximation of the distribution of the Maximum Likelihood (ML) estimator of the parameters in state space models. The first procedure proposed in this paper is parametric in the sense that it is based on resampling from the assumed distribution of the errors. The second procedure is based on resampling from the residuals of the estimated model and consequently, it does not assume any particular error distribution. We carry out Monte Carlo experiments to analyze the finite sample performance of our procedures which is compared with that of alternative procedures. We show that the biases of the PMSE proposed in this paper are smaller than those of the asymptotic procedures of Hamilton (1986) and the bootstrap PMSE procedure of Pfeffermann and Tiller (2005). The results are illustrated with simulated and real data highlighting the importance of incorporating the parameter uncertainty in empirical applications.

The rest of the paper is organized as follows. Section 2 describes the Kalman filter and illustrates with simulated data the biases incurred when estimating the PMSE of the estimated underlying states by running the filter with estimated parameters. We also briefly describe the asymptotic procedure of Hamilton (1986) and the bootstrap procedures proposed by Pfeffermann and Tiller (2005) to overcome these biases. In Section 3, we propose two new bootstrap procedures to obtain PMSE of the one-step-ahead estimator of the unobserved states that take into account the parameter uncertainty. Their finite sample properties are analyzed and compared with those of the standard Kalman filter, the asymptotic and previously available bootstrap PMSE. Section 4 contains an empirical application in which we estimate the uncertainty associated with the unobserved quarterly output gap, Non-Accelerating Inflation Rate of Unemployment (NAIRU), investment rate and core inflation in the US. Finally, Section 5 concludes the paper.

2. PMSE of Kalman filter estimates of states

Consider the following state space model

$$Y_t = Z_t\alpha_t + d_t + R_{1t}\varepsilon_t, \tag{1a}$$

$$\alpha_t = T_t\alpha_{t-1} + c_t + R_{2t}\eta_t, \quad t = 1, \ldots, T, \tag{1b}$$

where Y_t is an $N \times 1$ vector time series observed at time t, α_t is the $m \times 1$ vector of unobservable state variables, ε_t is a $k \times 1$ vector of independent white noise processes with zero mean and covariance matrix H_t and η_t is a $g \times 1$ vector of serially uncorrelated disturbances with zero mean and covariance matrix Q_t. The disturbances ε_t and η_t are uncorrelated with each