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# Comparison of estimates using record statistics from Weibull model: Bayesian and non-Bayesian approaches

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**Abstract**

This paper develops a Bayesian analysis in the context of record statistics values from the two-parameter Weibull distribution. The ML and the Bayes estimates based on record values are derived for the two unknown parameters and some survival time parameters e.g. reliability and hazard functions. The Bayes estimates are obtained based on a conjugate prior for the scale parameter and a discrete prior for the shape parameter of this model. This is done with respect to both symmetric loss function (squared error loss), and asymmetric loss function (linear-exponential (LINEX)) loss function. The maximum likelihood and the different Bayes estimates are compared via a Monte Carlo simulation study. A practical example consisting of real record values using the data from an accelerated test on insulating fluid reported by Nelson was used for illustration and comparison. Finally, Bayesian predictive density function, which is necessary to obtain bounds for predictive interval of future record is derived and discussed using a numerical example. The results may be of interest in a situation where only record values are stored.


*Keywords:* Record values; Symmetric and asymmetric loss functions; Maximum likelihood estimates; Bayesian estimation and prediction; Monte Carlo simulation

## 1. Introduction

The Weibull distribution is one of the most popular widely used models of failure time in life testing and reliability theory. The Weibull distribution has been shown to be useful for modeling and analysis of life time data in medical, biological and engineering sciences. Some applications of the Weibull distribution in forestry are given in Green et al. (1994). A great deal of research has been done on estimating the parameters of the Weibull distribution using both classical and Bayesian techniques, and a very good summary of this work can be found in Johnson et al. (1994). Recently, Hossain and Zimmer (2003) have discussed some comparisons of estimation methods for Weibull parameters using complete and censored samples.

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Record values and the associated statistics are of interest and important in many real life applications. In industry, many products fail under stress. For example, a wooden beam breaks when sufficient perpendicular force is applied to it, an electronic component ceases to function in an environment of too high temperature, and a battery dies under the stress of time. But the precise breaking stress or failure point varies even among identical items. Hence, in such experiments, measurements may be made sequentially and only the record values are observed. Thus, the number of measurements made is considerably smaller than the complete sample size. This "measurement saving" can be important when the measurements of these experiments are costly if the entire sample was destroyed. For more examples, see Gulati and Padgett (1994). There are also situations in which an observation is stored only if it is a record value. These include studies in meteorology, hydrology, seismology, athletic events and mining. In recent years, there has been much work on parametric and nonparametric inference based on record values. Among others are Resnick (1987), Nagaraja (1988), Ahsanullah (1993, 1995), Arnold et al. (1992, 1998), Gulati and Padgett (1994), Raqab and Ahsanullah (2001), and Raqab (2002). Sultan and Balakrishnan (1999) have developed some inferential method for the location and scale parameters of Weibull distribution. Ahmadi and Arghami (2001) discussed a comparison between the information (in Fisher's sense) contained in a set of $n$ upper record values with the Fisher information contained in $n$ iid observations from the original distribution. They showed that, for the Weibull Model, the upper record values contain more information than the same number of iid observations.

In this paper, we obtain and compare several techniques of estimation based on record statistics for the two unknown parameter of the Weibull distribution. As well as the survival time parameters, namely the hazard and Reliability functions. Section 2 contains some preliminaries. In Section 3, the Bayes estimators of the parameters, the reliability and hazard functions are derived. This is done using the conjugate prior on the scale parameter and discretizing the shape parameter to a finite number of values. The Bayes estimates are obtained using both the symmetric loss function (*s.e.l.*) and the asymmetric loss function (Varian's linear-exponential (LINEX)). A numerical example from an accelerated life-testing given by Nelson (1982) is used for illustration, and comparison is also given in Section 3. Results of a Monte Carlo simulation study conducted to evaluate the performance of these estimators compared to the MLEs in terms of mean squared error (MSE), are provided in Section 4. In Section 5, we provide Bayesian prediction interval for the future record with application examples. We conclude with some remarks and a brief summary of the results in Section 6.

## 2. Preliminaries

Let $X_1, X_2, X_3, \ldots$ a sequence of independent and identically distributed (iid) random variables with *cdf* $F(x)$ and *pdf* $f(x)$. Set $Y_n = \max(X_1, X_2, X_3, \ldots, X_n)$, $n \geqslant 1$, we say that $X_j$ is an upper record and denoted by $X_{U(j)}$ if $Y_j > Y_{j-1}$, $j > 1$. Assuming that $X_{U(1)}, X_{U(2)}, X_{U(3)}, \ldots, X_{U(n)}$ are the first $n$ upper record values arising from a sequence $\{X_i\}$ of iid Weibull variables with pdf

$$f(x) = \alpha\beta x^{\beta-1} \exp\left(-\alpha x^{\beta}\right), \quad x \geqslant 0, \quad \alpha, \beta > 0, \tag{1}$$

and cdf

$$F(x) = 1 - \exp\left(-\alpha x^{\beta}\right), \quad x \geqslant 0, \quad \alpha, \beta > 0, \tag{2}$$

where $\alpha$ and $\beta$ are scale and shape parameters, respectively. This version of the Weibull distribution separates the two parameters and often simplifies the algebra in the subsequent Bayesian manipulations.

The reliability function $R(t)$, and the hazard (instantaneous failure rate) function $H(t)$ at mission time $t$ for the Weibull distribution are given by

$$R(t) = \exp\left(-\alpha t^{\beta}\right), \quad t > 0, \tag{3}$$

and

$$H(t) = \alpha\beta t^{\beta-1}, \quad t > 0. \tag{4}$$