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Adaptive EWMA procedures for monitoring processes subject to linear drifts

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ABSTRACT

The conventional Statistical Process Control (SPC) techniques have been focused mostly on the detection of step changes in process means. However, there are often settings for monitoring linear drifts in process means, e.g., the gradual change due to tool wear or similar causes. The adaptive exponentially weighted moving average (AEWMA) procedures proposed by Yashchin (1995) have received a great deal of attention mainly for estimating and monitoring step mean shifts. This paper analyzes the performance of AEWMA schemes in signaling linear drifts. A numerical procedure based on the integral equation approach is presented for computing the average run length (ARL) of AEWMA charts under linear drifts in the mean. The comparison results favor the AEWMA chart under linear drifts. Some guidelines for designing AEWMA charts for detecting linear drifts are presented.

1. Introduction

Control charts are effective tools in statistical process control (SPC) for process monitoring and quality improvement. The applications of control charts extend far beyond industrial setting toward other areas such as biology, genetics, medicine, and finance (Montgomery, 2009). One of the most important problems in quality engineering is the detection of shifts in process means which occur in different ways. The shift may be a step change in the mean or a drift in the mean in a linear/nonlinear fashion. The step shift stays at the new level since its occurrence while the linear drift gradually increases/decreases over time in the mean. A great deal of attention has been devoted to the monitoring of step mean shifts while relatively less attention has been paid to the monitoring of linear drifts in the mean.

The output characteristics of many manufacturing processes exhibit drifts, and the drift in the mean can be positive or negative, linear or nonlinear. A typical example of positive drifts is tool wear, which describes the gradual failure of cutting tools due to regular operation. A tool wear-out leads to gradually increasing product dimension. An example of negative drifts includes a continuous clogging of a spray nozzle. In both examples, the output characteristics exhibit a drift instead of a step change in the process mean. As the drift in the process mean can cause significant losses in product quality, it is important to detect the drift as soon as it occurs. In practice, the linear model often serves as a good one for many drifts. For simplicity, this paper will limit discussions on the detection of linear drifts.

The conventional control charts for monitoring step shifts, including the Shewhart, cumulative sum (CUSUM), and exponentially weighted moving average (EWMA) control charts, have also been extended for monitoring linear drifts. For example, Davis and Woodall (1988) considered the Shewhart chart supplemented with run rules under a linear drift. Davis and Krehbiel (2002) investigated the performance of Shewhart and zone charts under linear trends. Rainer et al. (2001)

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suggested Shewhart-type UMP charts derived from the uniformly most powerful (UMP) test for monitoring linear drifts. Based on a modification of the Markov chain method developed by Brook and Evans (1972) and Bissell (1984) proposed methods for computing average run length (ARL) of CUSUM charts under linear drifts in the process mean. However, this procedure did not produce accurate ARL values. Gan (1992, 1996) further presented an accurate numerical method based on an integral equation for computing the ARL of CUSUM charts under linear trends. Koning and Does (2000)developed a CUSUM-type chart from the UMP test for the detection of linear trend. In addition to the use of Shewhart and CUSUM charts, Gan (1991) and Reynolds and Stoumbos (2001) considered the EWMA chart for detecting linear drifts. Other monitoring schemes for detecting linear drifts are given by Domangue and Patch (1991), Runger and Testik (2003), Fahmy and Elsayed (2006a,b) and Tseng et al. (2007).

Recently, Zou et al. (2009) made a comprehensive comparison among various control charts under linear drifts, including CUSUM, EWMA, generalized EWMA (GEWMA) and generalized likelihood ratio test (GLRT). They showed that the GLRT method can provide the best average performance at both small and large drifts in the process mean, and that the EWMA chart outperforms the CUSUM chart under linear drifts. Compared to the EWMA method, the GLRT method has slightly worse performance for detecting small linear drifts but much better performance for detecting large drifts. However, the GLRT procedure does not have recursive form and suffers from the computation load issue.

The literature on the efficiency and robustness of the EWMA chart for monitoring step mean shifts indicates that it is efficient in detecting small shifts in the process mean but not efficient for large mean shifts, as compared to the conventional Shewhart control charts. Yashchin (1995) investigated the estimation efficiency of the EWMA scheme in terms of an inertia function. He showed that the inertia increases as the magnitude of the mean shift increases. Therefore, the EWMA statistic with a small smoothing constant is not efficient in estimating abrupt mean changes of moderate and large magnitudes. This phenomenon has been referred to as the "inertia problem" (Woodall and Mahmoud, 2005). The inertia phenomenon occurs when the value of the EWMA statistic is on the lower side of the control limits but the shift occurs toward the opposite direction. In this case, if the smoothing constant is small, it takes a longer time for the EWMA statistic to exceed the control limits.

The inertia problem of the EWMA chart can be counteracted in part by using the combined Shewhart–EWMA chart (Lucas and Saccucci, 1990). However, the Shewhart–EWMA chart is not a smooth combination of the Shewhart principle and the EWMA scheme. To better overcome this problem, Yashchin (1995) proposed a smooth combination based on the adaptive EWMA (AEWMA) scheme. The underlying idea of the AEWMA procedure is to allocate the weight on past observations at each time step according to the magnitude of the estimation error. Unlike the conventional EWMA method, the smoothing constant used in the AEWMA scheme is no longer constant but varies over time. This weighting scheme has been widely discussed in the SPC literature recently. See, for example, Capizzi and Masarotto (2003), Shu (2008), Shu et al. (2008), Mahmoud and Zahran (2010) and Tseng et al. (2010).

Similarly, the EWMA chart for monitoring linear drifts also suffers from the inertia problem when the drift coefficient in the mean is large. In this article we extend the AEWMA scheme for monitoring linear drifts in the process mean. Markov chain, integral equation, and Monte Carlo simulations have been widely used to evaluate performance of a control chart. Similar to the approach of Gan (1991), an integral equation approach was developed to evaluate the performance of the AEWMA chart under linear drifts. This numerical approach allows for a quick analysis of the chart performance without running a large number of simulations. The AEWMA statistic can be expressed in an iterative way and thus can be viewed simpler in format than the GLRT procedure.

The rest of the paper is organized as follows. In Section 2, the AEWMA chart for monitoring linear drifts is introduced. In Section 3, the integral equation procedure for approximating the ARL of the AEWMA chart is developed. In Section 4, the approximation accuracy of the integral equation approach is evaluated. In Section 5, the performance of the AEWMA chart is compared with various control charts. In Section 6, a sequential design procedure is proposed to facilitate the implementation of AEWMA charts. Finally, some concluding remarks are given.

2. The AEWMA chart under linear drifts

Let $X_1, X_2, \ldots,$ be a sequence of observations collected at fixed intervals of time. When the process is in the state of statistical control, observations are assumed to be independently distributed from a normal distribution with a known mean μ_0 and known variance σ_0^2. After an unknown time point τ, the process mean is subject to a linear drift while the variance remains unchanged. The amount of drift is $\theta\sigma_0$ per unit time, where θ is unknown. In other words, the process mean at time t can be represented as

$$E(X_i) = \begin{cases} \mu_0, & i \le \tau \\ \mu_0 + \theta\sigma_0(i - \tau), & i > \tau. \end{cases}$$

Without loss of generality, we will assume the in-control process mean to be zero and the standard deviation to be one, i.e., $\mu_0 = 0$ and $\sigma_0^2 = 1$. Furthermore, we assume $\tau = 0$ to simplify the discussion. These assumptions are consistent with those made in Gan (1991). The ARL obtained assuming $\tau = 0$ has been referred to as the zero-state ARL while the ARL computed based on $\tau > 0$ has been called the steady-state ARL. Within our investigations, both the zero-state and steady-state results would provide qualitatively the same conclusion. For the sake of simplicity, we only consider the zero-state ARL performance in this paper while the steady-state ARL performance can be similarly analyzed.