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Factor estimation using MCMC-based Kalman filter methods

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ABSTRACT

An exact MCMC-based solution for the Kalman filter with Markov switching and GARCH components is proposed. To motivate the solution, an international equity market model incorporating common Markovian regimes and GARCH residuals in a persistent factor environment is considered. Given the intractable and approximate nature of the model's likelihood function, a Metropolis-in-Gibbs sampler with Bayesian features is constructed for estimation purposes. To accelerate the drawing procedure, approximations to the conditional density of the common component are also considered. The model is applied to equity data for 18 developed markets to derive global, European, and country-specific equity market factors.

1. Introduction

Exact and approximate MCMC-based solutions for the Kalman filter with Markov switching and GARCH components are proposed in this paper. The solutions extend King et al. (1994) contribution in deriving an approximate Kalman filter with GARCH volatilities, and effectively provide a first-time derivation of an exact Kalman filter in the presence of Markovian regimes and GARCH innovations. More generally, the solutions are applicable for any deterministic recursive volatility form, and may be used to jointly capture the persistence and volatility properties or characterise latent paths prevalent in financial or economic data.

The solutions are applied to estimate a unified factor model incorporating various facets of equity market return persistence, as well as co-movement possibilities in the time- and discrete regime-dependent contexts. The theoretical capacity for persistence and feedback in asset prices is discussed extensively in the behavioural finance literature (see, for example, De Long et al. (1990a,b), Daniel et al. (1998, 2001) and Hong and Stein (1999)). In turn, the specification of discrete regime-dependent volatility enables assessment of the effects of common shocks across the various volatility-regime structures, while asset-specific idiosyncratic volatility provides the capacity for deriving time and volatility-dependent measures of asset co-movement.

Although many papers have independently examined the behavioural and volatility components of equity returns (King et al., 1994; De Santis and Gerard, 1997; Aguilar and West, 2000; Grundy and Martin, 2001; Jegadeesh and Titman, 2001), the augmented Kalman filter setting is used in this paper to model the purported behavioural and volatility characteristics of equity market data.

To facilitate the estimation and obtain the requisite factors, a dynamic common factor model is constructed and applied to developed national equity market returns. In theory, the model may be exactly estimated using the Kalman filter with Markovian regimes and GARCH innovations. An exact likelihood function for such a filter cannot, however, be constructed (King et al., 1994; Kim and Nelson, 1998). Given the intractable and approximate nature of the adopted model's likelihood function, a Metropolis-in-Gibbs sampler is constructed to obtain exact Bayesian inferences for the dynamic factor model in the presence of persistence in the common and idiosyncratic components and their respective volatility structures.

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The paper is structured as follows. Section 2 describes the model structure. The estimation procedure is detailed in Section 3, while Section 4 considers approximations for accelerating the common factor drawing process. Section 5 applies the model to returns data for 18 national equity markets. The paper concludes with Section 6.

2. The model structure

The N-vector $r_t \in \mathbb{R}^N$ is treated as a function of K common factors:

$$r_t = \begin{bmatrix} C(L) & \mathrm{I}_N \end{bmatrix} \begin{bmatrix} \tilde{f}_t' & u_t' \end{bmatrix}' = C(L)\tilde{f}_t + u_t, \tag{1}$$

$$\Psi(L)\, u_t = \varepsilon_t = G_t^{1/2} z_t, \tag{2}$$

$$G_t = \mathrm{diag}\left(\sigma_t^2 = \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{2,t}^2 & \dots & \sigma_{N,t}^2 \end{bmatrix}'\right), \tag{3}$$

$$\sigma_{i,t}^2 = E\left(\sigma_i^2 | I_{t-1}\right) = \varpi_i + \sum_{p=1}^{P} \alpha_{i,p}\varepsilon_{i,t-p}^2 + \sum_{q=1}^{Q} \beta_{i,q}\sigma_{i,t-q}^2, \quad i = 1, 2, \dots, N, \tag{4}$$

where $C(L)$ is a polynomial in the lag operator L, $\tilde{f}_t$ is a K-dimensional vector, $\Psi(L) = \mathrm{I}_N - \Psi_1 L - \Psi_2 L^2 - \dots - \Psi_d L^d$ for finite d, $\Psi_j = \mathrm{diag}\left(\begin{bmatrix} \psi_{1,j} & \psi_{2,j} & \dots & \psi_{N,j} \end{bmatrix}\right)$ for $j = 1, 2, \dots, d$, z_t is a multivariate standard normal vector $z_t \sim iid\ MVN\left(\emptyset_{N,1}, \mathrm{I}_N\right)$ and $\emptyset_{a,b}$ is an $a \times b$ matrix of zeros. The GARCH specification in Eq. (4) is chosen given both its popularity and as a means of highlighting an exact estimator of King et al.'s (1994) model. The solutions in Sections 3 and 4, however, cater for any (boundedly) deterministic, recursive volatility form $\sigma_{i,t}^2$.

Each common factor is constructed as per the following:

$$\phi_k(L) f_{k,t} = \mu_{k,t} + e_{k,t}, \tag{5}$$

$$e_{k,t} \sim N\left(0, \sigma_{k,t}^2\right), \tag{6}$$

where $\phi_k(L) = 1 - \phi_{k,1}L - \dots - \phi_{k,N(k)}L^{N(k)}$, $\mu_{k,t} = \sum_{m=1}^{M(k)} \tilde{\mu}_{k,m} S_{k,m,t}$ and $\sigma_{k,t}^2 = \sum_{m=1}^{M(k)} \tilde{\sigma}_{k,m}^2 S_{k,m,t}$ for finite $N(k)$, $M(k)$, and $E\left(f_{k,a} u_b\right) = 0\, \forall a, b$. $S_{k,m,t}$ is a discrete latent variable taking the value unity if state $m = m(k)$ at time t, $m(k) \in M_k = \{m \in N : m \leq M(k) \in N\}$, and zero otherwise. The probability of state m prevailing is determined in accordance with the Markovian transition matrix:

$$Pr_k = \begin{bmatrix} P_{1,1,k} & \cdots & P_{M(k),1,k} \\ \vdots & \ddots & \vdots \\ P_{1,M(k),k} & \cdots & P_{M(k),M(k),k} \end{bmatrix},$$

where $Pr_k' 1_{M(k)} = 1_{M(k)}$ and $1_{M(k)}$ is an $M(k)$-dimensional column vector of ones, implying that $P_{m_1,m_2,k}$ represents the unconditional probability of a transition from state m_1 to state m_2 for the kth factor.

There are several motivations for this specification. Given $M(k) = 1 \forall k$, the model collapses to the popular King et al. (1994) model used to estimate common factors among national equity markets in the presence of factor persistence and GARCH volatility. A significant difference, however, is that in this paper the model is estimated exactly, thereby avoiding the biases involved in King et al.'s approximate solution (see, Tsiaplias (2007)). Alternatively, given $\sigma_{i,t}^2 = \sigma_i^2$, and $\phi = \psi = 1$, the specification collapses to the heteroscedastic common component form often used to estimate co-movement levels in the presence of common time-varying shocks (Diebold and Nerlove, 1989; Engle and Susmel, 1993; Kim and Nelson, 1998).

In the absence of these simplifying restrictions the specification jointly accommodates asset returns exhibiting persistent common and idiosyncratic factors, common Markovian jumps, and conditionally heteroscedastic volatility.[1] All the three properties are relevant in modelling asset returns. Traditionally, the use of lower frequency data ensured that the assumption of uncorrelated common and idiosyncratic factors ($\phi = \psi = 1$), in addition to simplifying the estimation process, was uncontroversial. In weekly or higher frequency data, however, such assumptions are likely to be violated as trading behaviour induces feedback effects (see, for example, Daniel et al. (1998, 2001).

Both the literature on jump models (Duffie et al., 2000; Eraker et al., 2003) and observation of actual financial market behaviour indicate that returns and volatility levels of a large number of markets frequently jump up or down together. The common Markovian jumps accommodate this behaviour. However, although jump effects may suffice in capturing heteroscedasticity at lower frequencies (Kim and Nelson, 1998), higher frequency data tend to exhibit residual heteroscedasticity in spite of jump effects.[2] This residual heteroscedasticity complicates the observed volatility structure and motivates the additional incorporation of asset-specific GARCH effects.

[1] Idiosyncratic volatility may also be modelled as a stochastic volatility process (see, for example, Kim et al. (1998)). GARCH idiosyncratic volatilities are chosen in this paper as a basis for comparing King et al.'s (1994) approximate solution for the Kalman filter with GARCH terms with the exact and approximate solutions to the Kalman filter with GARCH and Markov regime-switching terms presented here.

[2] This is observed empirically in the national equity market returns data used in this paper. Heteroscedasticity is adequately captured using regime switching for monthly returns. For weekly returns, however, residual heteroscedasticity is observed for all markets notwithstanding regime-switching volatility.