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Critical point computations for one-sided and two-sided pairwise comparisons of three treatment means

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ABSTRACT

This paper addresses the problem of critical point calculations for pairwise comparisons of three normal means. One-sided and two-sided pairwise comparisons are standard multiple comparisons procedures, and while tables of critical points exist for balanced situations with equal sample sizes, only limited tables of critical points exist for unbalanced cases. A new algorithm is developed in this paper using elementary methods which allows the critical points to be found in all situations using only a one-dimensional numerical integration. Programs have been developed to implement the algorithm which will allow experimenters to easily find the required critical points and p-values.

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1. Introduction

Consider the unbalanced one-way analysis of variance model

$$X_{ij} = \mu_i + \epsilon_{ij}, \quad 1 \le i \le 3,\ 1 \le j \le n_i$$

where ϵ_{ij} are independent $N(0, \sigma^2)$ random variables. Let $\bar{X}_i$, $1 \le i \le 3$, be the ith sample mean based upon n_i observations, and let S^2 be an unbiased estimate of σ^2 distributed independently of the $\bar{X}_i$ as $S^2 \sim \sigma^2 \chi^2_\nu/\nu$ for some degrees of freedom ν. Usually the mean square error in the analysis of variance will be used as the estimate S^2 with $\nu = \sum_{i=1}^{3} n_i - 3$.

Suppose that the data represent information on three treatments which can be assumed to satisfy the simple ordering $\mu_1 \le \mu_2 \le \mu_3$. Then the set of $1-\alpha$ one-sided simultaneous confidence intervals

$$\begin{aligned}
\mu_3 - \mu_1 &\in \left(\bar{X}_3 - \bar{X}_1 - h_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_3}}, \infty\right)\\
\mu_3 - \mu_2 &\in \left(\bar{X}_3 - \bar{X}_2 - h_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_2}+\frac{1}{n_3}}, \infty\right)\\
\mu_2 - \mu_1 &\in \left(\bar{X}_2 - \bar{X}_1 - h_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}, \infty\right)
\end{aligned} \tag{1}$$

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provides useful inferences on the possible differences between the treatment means, as discussed in Hayter (1990). The critical points $h_{\alpha,\mathbf{n},\nu}$ with $\mathbf{n} = (n_1, n_2, n_3)$ are chosen so that these confidence intervals have a confidence level of exactly $1-\alpha$, and the computation of such critical points is the focus of this paper.

Similarly, when no ordering of the treatment means is assumed, the set of $1-\alpha$ two-sided simultaneous confidence intervals

$$\begin{aligned}
\mu_3-\mu_1 &\in \left(\bar{X}_3-\bar{X}_1-q_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_3}},\bar{X}_3-\bar{X}_1+q_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_3}}\right)\\
\mu_3-\mu_2 &\in \left(\bar{X}_3-\bar{X}_2-q_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_2}+\frac{1}{n_3}},\bar{X}_3-\bar{X}_2+q_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_2}+\frac{1}{n_3}}\right)\\
\mu_2-\mu_1 &\in \left(\bar{X}_2-\bar{X}_1-q_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}},\bar{X}_2-\bar{X}_1+q_{\alpha,\mathbf{n},\nu}\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right)
\end{aligned} \tag{2}$$

proposed by Tukey (unpublished manuscript) and Kramer (1956) again provides useful inferences on the possible differences between the treatment means. Even though the exact Studentized range critical point $q_{\alpha,\mathbf{n},\nu}$ is known to be smaller than the corresponding balanced (equal sample size) critical point (see Hayter (1984)), it is useful to be able to calculate the exact critical point, and that is again the focus of this paper.

The construction of the exact one-sided critical points for unbalanced situations was considered in Hayter (1992) and Hayter and Liu (1996) while the construction of the exact two-sided critical points for unbalanced situations was considered in Spurrier and Isham (1985) and Uusipaikka (1985), and some tables were provided in each case. However, those papers used either algorithms that required multi-dimensional numerical integrations or sophisticated mathematics. The algorithms developed in this paper have the advantage that they only require elementary mathematics and the evaluation of a one-dimensional numerical integration. Furthermore, Matlab routines have been developed (available from the second author) that will allow experimenters the easy evaluation of these critical points in all situations. An additional advantage of the work provided in this paper is that it illustrates how similar types of problems (such as inferences on sets of contrasts of three normally distributed estimates) can be dealt with using the same idea and accomplished with only one-dimensional integral computations.

Other methods are available for computing the critical constants. For example, Genz and Bretz (1999, 2002), Somerville (1998, 1999) and Somerville and Bretz (2001) propose numerical integration methods to calculate multivariate normal and t probabilities. However, these methods all involve Monte Carlo simulations rather than exact numerical computations, even though they are much more accurate than crude simulation methods. It should also be noted that the methods discussed in this paper are generally applicable to a set of independently distributed normal estimates, which may arise from more complicated models such as two-way designs which are discussed in Cheung and Chan (1996).

Both the one-sided and two-sided procedures can be used to test the null hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$. A size α test rejects when at least one of the $1-\alpha$ level confidence intervals does not contain zero. For the two-sided case a p-value for the null hypothesis can be obtained as one minus the confidence level at which one confidence interval has zero as an endpoint and the other two confidence intervals contain zero. This is also true for the one-sided case when the ordered differences of the sample means $\bar{X}_3-\bar{X}_1, \bar{X}_3-\bar{X}_2$, and $\bar{X}_2-\bar{X}_1$, are all positive. Our Matlab routine also provides these p-values and Fig. 1 provides a flowchart of the structure of the program.

In the remainder of this paper, Section 2 contains the development of the algorithm for the one-sided case while Section 3 contains the development of the algorithm for the two-sided case. Section 4 contains an illustrative example.

2. The one-sided case

Define

$$\begin{aligned}
A(h) = \Bigg\{&(\bar{X}_1,\bar{X}_2,\bar{X}_3) : (\bar{X}_3-\mu_3)-(\bar{X}_1-\mu_1) \le h\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_3}},\\
&(\bar{X}_3-\mu_3)-(\bar{X}_2-\mu_2) \le h\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_2}+\frac{1}{n_3}}, (\bar{X}_2-\mu_2)-(\bar{X}_1-\mu_1) \le h\frac{S}{\sqrt{2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\Bigg\}
\end{aligned} \tag{3}$$

so that $h_{\alpha,\mathbf{n},\nu}$ is the solution to

$$\mathbb{P}\left\{(\bar{X}_1,\bar{X}_2,\bar{X}_3) \in A(h_{\alpha,\mathbf{n},\nu})\right\} = 1-\alpha.$$

With

$$Z_1 = \frac{(\bar{X}_3-\mu_3)-(\bar{X}_1-\mu_1)}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_3}}} \sim N(0,1)$$

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