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Modelling long-memory volatilities with leverage effect: A-LMSV versus FIEGARCH

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Abstract

A new stochastic volatility model, called A-LMSV, is proposed to cope simultaneously with leverage effect and long-memory in volatility. Its statistical properties are derived and compared with the properties of the FIEGARCH model. It is shown that the dependence of the autocorrelations of squares on the parameters measuring the asymmetry and the persistence is different in both models. The kurtosis and autocorrelations of squares do not depend on the asymmetry in the A-LMSV model while they increase with the asymmetry in the FIEGARCH model. Furthermore, the autocorrelations of squares increase with the persistence in the A-LMSV model and decrease in the FIEGARCH model. On the other hand, if the correlation between returns and future volatilities is negative, the autocorrelations of absolute returns increase with the magnitude of the asymmetry in the FIEGARCH model while they decrease in the A-LMSV model. Finally, the cross-correlations between squares and original observations are, in general, larger in absolute value in the FIEGARCH model than in the A-LMSV model. The results are illustrated by fitting both models to represent the dynamic evolution of volatilities of daily returns of the S&P500 and DAX indexes.

Keywords: Autocorrelations of squares and of absolute values; Conditional heteroscedasticity; Kurtosis; EMM estimator

1. Introduction

One of the main empirical characteristics of financial returns is the dynamic evolution of their volatilities. There are two important properties that characterized this evolution. First of all, power transformations of absolute returns have significant autocorrelations which decay toward zero slower than in a short-memory process. Many authors have argued that this pattern of the sample autocorrelations suggests that the volatilities of financial returns can be represented by long-memory processes; see Ding et al. (1993) and Lobato and Savin (1998) among many others. The second property that characterizes volatilities is their asymmetric response to positive and negative returns. This property, known as leverage effect, was first described by Black (1976).

There are two main families of econometric models proposed to represent the dynamic evolution of volatilities. The ARCH-type models are mainly characterized by specifying the volatility as a function of powers of past absolute returns and, consequently, the volatility can be observed one-step ahead. On the other hand, stochastic volatility (SV) models specify the volatility as a latent variable that is not directly observable. There have been several proposals of ARCH-type models that represent simultaneously leverage effect and long-memory. For example, Hwang (2001) generalizes the

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long-memory FIGARCH model of Baillie et al. (1996) to represent the leverage effect. However, Davidson (2004) shows that the FIGARCH model has the unpleasant property that the persistence of shocks to volatility decreases as the long-memory parameter increases. A similar conclusion appears in Zaffaroni (2004) who shows that the FIGARCH model cannot generate autocorrelations of squares with long-memory. Finally, Ruiz and Pérez (2003) showed that the model proposed by Hwang (2001) has identification problems. Consequently, in this paper, we focus on the fractionally integrated EGARCH (FIEGARCH) model proposed by Bollerslev and Mikkelsen (1996) which extends the asymmetric EGARCH model of Nelson (1991) to long-memory. Following the arguments of He et al. (2002) for the short-memory EGARCH model, we derive the kurtosis, autocorrelations of squares and absolute observations and the cross-correlations between returns and powers of absolute returns of the FIEGARCH$(1, d, 0)$ model with Gaussian errors.

Alternatively, in the context of SV models, Harvey (1998) and Breidt et al. (1998) have independently proposed long-memory stochastic volatility (LMSV) models in which the underlying log-volatility is modelled as an ARFIMA process. On the other hand, Harvey and Shephard (1996) propose to model the leverage effect of the short-memory SV model by introducing correlation between the noises of the level and volatility equations. Recently, Yu et al. (2006) have also proposed an extension of this asymmetric SV model in which a Box–Cox transformation of the volatility follows an AR(1) process. This model encompasses several popular SV models as special cases. However, the statistical properties of the new model are still unknown. So et al. (2002) have proposed a threshold SV model which is also able to represent the leverage effect. In this paper, we focus on the proposal of Harvey and Shephard (1996) and propose the Asymmetric LMSV (A-LMSV) model that generalizes the LMSV model to cope with leverage effect. Assuming that the log-volatility is an ARFIMA$(1, d, 0)$ model and Gaussianity of the errors, we derive the statistical properties of the new model and compare them with the properties of the FIEGARCH model. We show that both models explain in different ways the kurtosis and correlations of absolute and squared returns. However, the cross-correlations between returns and powers of absolute returns behave in a similar fashion.

The rest of the paper is organized as follows. The description of the statistical properties of the A-LMSV$(1, d, 0)$ model is done in Section 2. In Section 3, we derive the properties of the FIEGARCH$(1, d, 0)$ model and compare them with the properties of the A-LMSV model. Section 4 contains an empirical illustration by fitting both models to daily financial returns of the S&P500 and the DAX indexes. Section 5 concludes the paper.

2. Asymmetric LMSV models

The LMSV model, proposed independently by Harvey (1998) and Breidt et al. (1998), extends the stochastic volatility model of Taylor (1982) by assuming that the volatility follows a weakly stationary fractional integrated process. Therefore, the LMSV model captures the long-memory property often observed in the powers of absolute returns. In this section, we extend the LMSV model to represent the asymmetric response of volatility to positive and negative returns. Following Taylor (1994) and Harvey and Shephard (1996), this asymmetry is introduced by allowing the disturbances of the level and volatility equations to be correlated. In this paper, we consider that the log-volatility is specified as an ARFIMA$(1, d, 0)$ process. In this case, the A-LMSV model is given by

$$y_t = \sigma_* \sigma_t \varepsilon_t,$$
$$(1 - \phi L)(1 - L)^d \log \sigma_t^2 = \eta_t, \quad (1)$$

where y_t is the return at time t and σ_t is its volatility. The parameter σ_* is a scale parameter and L is the lag operator such that $Lx_t = x_{t-1}$. The disturbances $(\varepsilon_t, \eta_{t+1})'$ are assumed to have the following bivariate normal distribution:

$$\begin{pmatrix} \varepsilon_t \\ \eta_{t+1} \end{pmatrix} \sim NID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \delta\sigma_\eta \\ \delta\sigma_\eta & \sigma_\eta^2 \end{pmatrix} \right), \quad (2)$$

where δ, the correlation between ε_t and η_{t+1}, induces correlation between the returns, y_t, and the variations of the volatility one period ahead, $\sigma_{t+1} - \sigma_t$. The dynamic properties of the symmetric LMSV model are described by Ghysels et al. (1996). In particular, the stationarity of y_t depends on the stationarity of the log-volatility, $h_t = \log \sigma_t^2$. Therefore, if $|\phi| < 1$ and $d < 0.5$, y_t is stationary. In this case, they show that the variance and kurtosis of y_t are given by

$$Var(y_t) = \sigma_*^2 \exp\left\{ \frac{\sigma_h^2}{2} \right\} \quad (3)$$