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Generalized exponential distribution: Bayesian estimations[☆]

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Abstract

Recently two-parameter generalized exponential distribution has been introduced by the authors. In this paper we consider the Bayes estimators of the unknown parameters under the assumptions of gamma priors on both the shape and scale parameters. The Bayes estimators cannot be obtained in explicit forms. Approximate Bayes estimators are computed using the idea of Lindley. We also propose Gibbs sampling procedure to generate samples from the posterior distributions and in turn computing the Bayes estimators. The approximate Bayes estimators obtained under the assumptions of non-informative priors, are compared with the maximum likelihood estimators using Monte Carlo simulations. One real data set has been analyzed for illustrative purposes.

Keywords: Bayes estimator; Maximum likelihood estimator; Gamma distribution; Log-concave density function; Squared error loss function; Posterior density function

1. Introduction

Recently the authors proposed (Gupta and Kundu, 1999) the two-parameter generalized exponential distribution (GE) as an alternative to the gamma and Weibull distributions and studied its different properties. Some of the recent references on GE distribution are Raqab (2002), Raqab and Ahsanullah (2001), Zheng (2002), Sarhan (2007), Gupta and Kundu (2007), Kundu and Gupta (2007) and the references cited there.

The two-parameter GE has the following density function:

$$f(x;\alpha,\lambda)=\alpha\lambda(1-\mathrm{e}^{-\lambda x})^{\alpha-1}\mathrm{e}^{-\lambda x},\quad x>0, \tag{1}$$

here $\alpha, \lambda > 0$ are the shape and scale parameters, respectively. The main aim of this paper is to consider the Bayesian analysis of the GE distribution and compare their performances with the classical ones. Since both α and λ are non-negative therefore, it is quite natural to assume the gamma priors on α and λ, although they are not the conjugate priors. In many practical situations, the information about the shape and scale of the sampling distribution is available in an independent manner, see Basu et al. (1999). Therefore, here it is assumed that the parameters α and λ are independent *a priori*.

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In this paper we mainly consider the squared error loss function. It is observed that the Bayes estimators cannot be expressed in explicit forms and they can be obtained by two dimensional numerical integrations only. We use the idea of Lindley to compute the approximate Bayes estimators of the unknown parameters and it is observed that the approximation works quite well. We compute the approximate Bayes estimators under the assumption of non-informative priors and compare them with the maximum likelihood estimators (MLEs) by Monte Carlo simulations. We also propose Markov Chain Monte Carlo (MCMC) techniques to generate samples from the posterior distributions and in turn computing the Bayes estimators. The posterior density functions match quite well with the histograms of the samples obtained by MCMC methods.

It should be mentioned that Jaheen (2004) and Raqab and Madi (2005) also considered the Bayesian inferences of the unknown parameter(s) of the GE distribution. Jaheen (2004) considered the empirical Bayes estimate of the shape parameter when the scale parameter is known. Raqab and Madi (2005) considered the case when both the parameters are unknown, but their approaches and the emphasis are quite different than the present paper. Moreover, in this paper we have observed that our method can be extended to a more general class of distributions, for example proportional reversed hazard models or for exponentiated Weibull distribution. However, it is not clear how to extend the results of Jaheen (2004) and Raqab and Madi (2005) in a more general situation.

The rest of the paper is organized as follows. In Section 2, we propose the Bayes estimators of the unknown parameters. The approximate Bayes estimators are also considered in Section 2. One data set has been analyzed in Section 3. Numerical results are provided in Section 4. In Section 5 we briefly mentioned how to extend our results for more general classes of distributions and finally conclusions appear in Section 6.

2. Bayes estimation of the unknown parameter(s)

In this section we consider the Bayes estimation of the unknown parameter(s). When both are unknown, it is assumed that α and λ have the following gamma prior distributions;

$$\pi_1(\lambda) \propto \lambda^{b-1} e^{-a\lambda}, \quad \lambda > 0, \tag{2}$$

$$\pi_2(\alpha) \propto \alpha^{d-1} e^{-c\alpha}, \quad \alpha > 0. \tag{3}$$

Here all the hyper parameters a, b, c, d are assumed to be known and non-negative.

Suppose $\{x_1, \ldots, x_n\}$ is a random sample from $GE(\alpha, \lambda)$, then based on the likelihood function of the observed data;

$$l(data|\alpha, \lambda) = \alpha^n \lambda^n e^{-\lambda \sum_{i=1}^n x_i} \prod_{i=1}^n (1 - e^{-\lambda x_i})^{\alpha - 1}, \tag{4}$$

the joint posterior density function of α and λ can be written as

$$l(\alpha, \lambda|data) = \frac{l(data|\alpha, \lambda)\pi_1(\lambda)\pi_2(\alpha)}{\int_0^\infty \int_0^\infty l(data|\alpha, \lambda)\pi_1(\lambda)\pi_2(\alpha)\, d\alpha\, d\lambda}. \tag{5}$$

Therefore, the Bayes estimator of any function of α and λ, say $g(\alpha, \lambda)$ under the squared error loss function is

$$\hat{g}_B = E_{\alpha,\lambda|data}(g(\alpha, \lambda)) = \frac{\int_0^\infty \int_0^\infty g(\alpha, \lambda) l(data|\alpha, \lambda)\pi_1(\lambda)\pi_2(\alpha)\, d\alpha\, d\lambda}{\int_0^\infty \int_0^\infty l(data|\alpha, \lambda)\pi_1(\lambda)\pi_2(\alpha)\, d\alpha\, d\lambda}. \tag{6}$$

It is not possible to compute analytically (6) in this case. Two approaches are used here namely (a) Lindley's approximation; (b) Markov Chain Monte Carlo (MCMC) to approximate (6). Note that when one parameter is known, the Bayes estimator of any function of the other parameter also can be written similarly as the ratio of two integrals such as (6).

2.1. Lindley's approximation

Lindley (1980) proposed his procedure to approximate the ratio of the two integrals such as (6). This has been used by several authors to obtain the approximate Bayes estimators. For details see Lindley (1980) or Press (2001). Based