Computational Statistics and Data Analysis 58 (2013) 352–367

Contents lists available at SciVerse ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

The compound class of extended Weibull power series distributions

Rodrigo B. Silva *, Marcelo Bourguignon, Cícero R.B. Dias, Gauss M. Cordeiro

Universidade Federal de Pernambuco, Departamento de Estatística, Cidade Universitária, 50740-540 Recife, PE, Brazil

ARTICLE INFO

Article history:
Received 2 March 2012
Received in revised form 7 September 2012
Accepted 14 September 2012
Available online 25 September 2012

Keywords:
Extended Weibull distribution
Extended Weibull power series distribution
Order statistic
Power series distribution

ABSTRACT

We introduce a general method for obtaining more flexible new distributions by compounding the extended Weibull and power series distributions. The compounding procedure follows the same set-up carried out by Adamidis and Loukas (1998) and defines 68 new sub-models. The new class of generated distributions includes some well-known mixing distributions, such as the Weibull power series (Morais and Barreto-Souza, 2011) and exponential power series (Chahkandi and Ganjali, 2009) distributions. Some mathematical properties of the new class are studied including moments and the generating function. We provide the density function of the order statistics and their moments. The method of maximum likelihood is used for estimating the model parameters. Special distributions are investigated. We illustrate the usefulness of the new distributions by means of two applications to real data sets.

© 2012 Elsevier B.V. All rights reserved.

1. Introduction

The modeling and analysis of lifetimes is an important aspect of statistical work in a wide variety of scientific and technological fields. Several distributions have been proposed in the literature to model lifetime data by compounding some useful lifetime distributions. Adamidis and Loukas (1998) introduced a two-parameter exponential geometric (EG) distribution by compounding the exponential and geometric distributions. In a similar manner, the exponential Poisson (EP) and exponential logarithmic (EL) distributions were introduced and studied by Kus (2007) and Tahmasbi and Rezaei (2008), respectively. Recently, Chahkandi and Ganjali (2009) have proposed the exponential power series (EPS) family of distributions, which contains as special cases these distributions. Barreto-Souza et al. (2010) and Lu and Shi (2011) introduced the Weibull geometric (WG) and Weibull Poisson (WP) distributions which naturally extend the EG and EP distributions, respectively. In a very recent paper, Morais and Barreto-Souza (2011) defined the Weibull power series (WPS) class of distributions which includes as sub-models the EPS distributions. The WPS distributions can have an increasing, decreasing and upside down bathtub failure rate function. The generalized exponential power series (GEPS) distributions were proposed by Mahmoudi and Jafari (2012) following the same approach developed by Morais and Barreto-Souza (2011). Another recent compounded distribution can be found in Cancho et al. (2011, 2012) that introduced the Poisson exponential (PE) and geometric Birnbaum–Saunders (GBS) distributions and Barreto-Souza and Bakouch (2012) who defined the Poisson Lindley (PL) distribution. Further, Louzada et al. (2011) and Cordeiro et al. (2012) proposed the complementary exponential geometric (CEG) and the exponential COM Poisson (ECOMP) distributions.

The Weibull distribution was one of the earliest and most popular model for failure times. In recent years, many authors have proposed generalizations of the Weibull model based on extended types of failure of a system. In the context of modeling random strength of brittle materials and failure times, Gurvich et al. (1997) proposed an extended Weibull (EW)

* Corresponding author.
E-mail addresses: rodrigobs29@gmail.com (R.B. Silva), m.p.bourguignon@gmail.com (M. Bourguignon), cicerorafael@gmail.com (C.R.B. Dias), gauss@de.ufpe.br (G.M. Cordeiro).

0167-9473/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.csda.2012.09.009

family of distributions. Nadarajah and Kotz (2005) and Pham and Lai (2007) presented much more than twenty useful distributions in their family. Its cumulative distribution function (cdf) is given by

$$G(x;\ \alpha, \boldsymbol{\xi}) = 1 - \mathrm{e}^{-\alpha H(x;\ \boldsymbol{\xi})}, \quad x > 0,\ \alpha > 0, \tag{1}$$

where $H(x;\ \boldsymbol{\xi})$ is a non-negative monotonically increasing function which depends on a parameter vector $\boldsymbol{\xi}$. The corresponding probability density function (pdf) becomes

$$g(x;\ \alpha, \boldsymbol{\xi}) = \alpha\, h(x;\ \boldsymbol{\xi})\, \mathrm{e}^{-\alpha H(x;\ \boldsymbol{\xi})}, \quad x > 0,\ \alpha > 0, \tag{2}$$

where $h(x;\ \boldsymbol{\xi})$ is the first derivative of $H(x;\ \boldsymbol{\xi})$. Many well-known models are special cases of Eq. (1) such as:

(i) $H(x; \boldsymbol{\xi}) = x$ gives the exponential distribution;
(ii) $H(x; \boldsymbol{\xi}) = x^2$ yields the Rayleigh distribution (Burr type-X distribution);
(iii) $H(x; \boldsymbol{\xi}) = \log(x/k)$ leads to the Pareto distribution;
(iv) $H(x; \boldsymbol{\xi}) = \beta^{-1}[\exp(\beta x) - 1]$ gives the Gompertz distribution.

We emphasize that several other distributions could be re-written in form (1) (see some examples in Nadarajah and Kotz, 2005; Pham and Lai, 2007). In this paper, we define the extended Weibull power series (EWPS) class of univariate distributions obtained by compounding the extended Weibull and power series distributions. The compounding procedure follows the key idea of Adamidis and Loukas (1998) or, more generally, by Chahkandi and Ganjali (2009) and Morais and Barreto-Souza (2011). The new class of distributions includes as special models the WPS distributions, which in turn extends the EPS distributions and defines 68 (17 × 4) new sub-models as special cases. The hazard function of the proposed class can be decreasing, increasing, bathtub and upside down bathtub. We are motivated to introduce the EWPS distributions because of the wide usage of (1) and the fact that the current generalization provides means of its continuous extension to still more complex situations.

This paper is organized as follows. In Section 2, we define the EWPS class of distributions and demonstrate that there are many existing models which can be deduced as special cases of the proposed unified model. In Section 3, we provide general properties of the EWPS class including the density, survival and hazard rate functions, some useful expansions, quantiles, ordinary and incomplete moments, generating function, order statistics and their moments, reliability and average lifetime. Estimation of the parameters by maximum likelihood is investigated in Section 4. In Section 5, we present suitable constraints leading to the maximum entropy characterization of the new class. Three special models of the proposed class are studied in Section 6. Applications to two real data sets are presented in Section 7. Some concluding remarks are addressed in Section 8.

2. The new class

Let N be a discrete random variable having a power series distribution (truncated at zero) with probability mass function

$$p_n = P(N = n) = \frac{a_n\, \theta^n}{C(\theta)}, \quad n = 1, 2, \ldots, \tag{3}$$

where a_n depends only on n, $C(\theta) = \sum_{n=1}^{\infty} a_n\, \theta^n$ and $\theta > 0$ is such that $C(\theta)$ is finite. The proposed class of distributions can be derived as follows. Given N, let $X_1, \ldots, X_N$ be independent and identically distributed (iid) random variables following (1). Table 1 summarizes some power series distributions (truncated at zero) defined according to (3) such as the Poisson, logarithmic, geometric and binomial distributions. Let $X_{(1)} = \min\{X_i\}_{i=1}^{N}$. The conditional cumulative distribution of $X_{(1)}|N = n$ is given by

$$G_{X_{(1)}|N=n}(x) = 1 - \mathrm{e}^{-n\alpha H(x;\boldsymbol{\xi})},$$

i.e., $X_{(1)}|N = n$ has the general class of distributions (1) with parameters $n\alpha$ and $\boldsymbol{\xi}$ based on the same $H(x;\ \boldsymbol{\xi})$ function. Hence, we obtain

$$P(X_{(1)} \le x, N = n) = \frac{a_n\, \theta^n}{C(\theta)} \left[1 - \mathrm{e}^{-n\alpha H(x;\boldsymbol{\xi})}\right], \quad x > 0,\ n \ge 1.$$

The EWPS class of distributions is defined by the marginal cdf of $X_{(1)}$:

$$F(x; \theta, \alpha, \boldsymbol{\xi}) = 1 - \frac{C(\theta\, \mathrm{e}^{-\alpha H(x;\boldsymbol{\xi})})}{C(\theta)}, \quad x > 0. \tag{4}$$

We provide at least four motivations for the EWPS class of distributions, which can be applied in some interesting situations as follows:

1. Time to the first failure. Suppose that the failure of a device occurs due to the presence of an unknown number N of initial defects of same kind, which can be identifiable only after causing failure and are repaired perfectly. Define by X_i

Download English Version:

https://daneshyari.com/en/article/417509

Download Persian Version:

https://daneshyari.com/article/417509

Daneshyari.com