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Bayesian estimation of generalized hyperbolic skewed student GARCH models

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ABSTRACT

Efficient posterior simulators for two GARCH models with generalized hyperbolic disturbances are presented. The first model, GHt-GARCH, is a threshold GARCH with a skewed and heavy-tailed error distribution; in this model, the latent variables that account for skewness and heavy tails are identically and independently distributed. The second model, ODLV-GARCH, is formulated in terms of observation-driven latent variables; it automatically incorporates a risk premium effect. Both models nest the ordinary threshold t-GARCH as a limiting case. The GHt-GARCH and ODLV-GARCH models are compared with each other and with the threshold t-GARCH using five publicly available asset return data sets, by means of Bayes factors, information criteria, and classical forecast evaluation tools. The GHt-GARCH and ODLV-GARCH models both strongly dominate the threshold t-GARCH, and the Bayes factors generally favor GHt-GARCH over ODLV-GARCH. A Markov switching extension of GHt-GARCH is also presented. This extension is found to be an empirical improvement over the single-regime model for one of the five data sets.

1. Introduction

The conditional distributions of asset returns are well-known to be leptokurtic. They are also known to exhibit the "leverage effect": a negative past innovation on asset returns tends to increase the current volatility. The leptokurticity justifies the introduction of heavy-tailed (e.g. Student or GED) disturbances in the GARCH class of models, and the "leverage effect" has motivated consideration of asymmetric extensions of the basic GARCH variance function introduced by Bollerslev (1986). An example of such an extension is proposed by Glosten et al. (1993). An excellent survey of ARCH and GARCH models can be found in Bollerslev et al. (1994).

There can be no certainty that the GARCH variance function will capture all the asymmetry present in an asset return distribution, even when this function incorporates a leverage effect. Presumably for this reason, several GARCH models with skewed error distributions can be found in the recent literature. Examples are Mittnik and Paolella (2000), Giot and Laurent (2003), Bauwens and Laurent (2005), Aas and Haff (2006) and Dark (2010). Aas and Haff (2006) provide additional references. However, none of these contributions propose a Bayesian treatment of the estimation problem: the models are estimated by maximum likelihood or quasi-maximum likelihood. This absence of Bayesian treatments is unfortunate, since the Bayesian paradigm offers a natural way of taking both parameter uncertainty and model uncertainty into account. Geweke and Amisano (2010) show that this is important in the context of forecast evaluation; it is therefore also likely to have an impact in the context of value at risk or expected shortfall estimation.

On the other hand, the Bayesian Markov chain Monte Carlo (MCMC) estimation of t-GARCH models with symmetric errors, but possibly asymmetric variance functions is now well-established; an efficient method, based on the previous

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contribution of Nakatsuma (2000), is fully described in Ardia (2008, chapter 5). This method relies on the fact that a Student-*t* variate can be represented as a Normal variate with stochastic variance; see, e.g., Geweke (1993). It is indeed this fact which allows an efficient Bayesian posterior simulator to be designed, using the technique of data augmentation.

As shown by Barndorff-Nielsen (1977), a Normal distribution can also be extended by taking *both* moments of the Normal to be functions of an inverted Gamma variate. The resulting distribution is leptokurtic and skewed. It is known as the Generalized Hyperbolic (GH) distribution, and has been extensively discussed by Prause (1999); see also Paolella (2007, chapter 9). However, empirical applications of the GH distribution have been few, perhaps due to the fact that its parameters can be difficult to identify in general.

Recently, however, Aas and Haff (2006) investigated a special case of the GH distribution that may considerably alleviate the identification problem mentioned above. It has the advantage of nesting the ordinary Student-*t* as a limiting case, and can therefore be called the Generalized Hyperbolic Skewed Student-*t* (GHSST). Aas and Haff (2006) show that the GHSST can exhibit unequal thickness in both tails, contrary to other skewed extensions of the Student-*t*, and argue that this offers more flexibility.

In this paper, we will propose an efficient MCMC posterior simulator, based on data augmentation, that can be used with models having a GHSST error distribution. It will be applied to two GARCH formulations. The first one (called GHt-GARCH for short) is a threshold version of the GARCH model in Aas and Haff (2006). In this first model, the inverted Gamma latent variables are identically distributed. In the second model, by contrast, these latent variables can be interpreted as stochastic volatilities, since their conditional distributions depend on past observations. The second model can therefore be called an "observation-driven stochastic volatility model" in the sense of Barndorff-Nielsen (1997), and does not appear to have been estimated before by any method. In order to avoid possible confusion between this model and the state-space volatility models that have been proposed in the literature, we will refer to this second model by the acronym ODLV-GARCH, where ODLV stands for "observation-driven latent variables".

Both the ODLV-GARCH and GHt-GARCH models nest the ordinary *t*-GARCH as a limiting case. So, they belong to a different class than the state-space formulation used by Nakajima and Omori (2012), who used the GHSST distribution in conjunction with an evolution equation implying lognormal volatilities.

The *t*-GARCH, ODLV-GARCH, and GHt-GARCH models will be compared using asset return data, by means of Bayes factors, Bayes information criteria, and classical forecast evaluation tools.

An outline of the paper is as follows. In Section 2, we state the GHt-GARCH model. Section 3 discusses the ODLV-GARCH model and the differences between this model and GHt-GARCH. Section 4 describes the posterior simulator. Section 5 presents empirical results based on five publicly available asset return data sets. Section 6 discusses a Markov switching extension, and Section 7 concludes the paper. In the Appendix, we discuss at length an efficient algorithm for drawing the latent variables used in data augmentation.

2. The GHt-GARCH model

This section will present an AR(1)-GARCH model with an asymmetric variance function and the skewed heavy-tailed error distribution discussed in Aas and Haff (2006). We model the log-return y_t of an asset at time t as

$$y_t = \phi_1 + \phi_2 y_{t-1} + u_t \quad \text{for } t = 1, \ldots, T, \tag{1}$$

where $u_t = \sigma_t \eta_t$ and η_t follows a GHSST distribution with zero expectation and unit variance. The variance equation is an asymmetric GARCH model of the type proposed by Glosten et al. (1993):

$$\sigma_t^2 = \alpha_0^* + [\alpha_1^* \mathcal{I}(u_{t-1} \geq 0) + \alpha_2^* \mathcal{I}(u_{t-1} < 0)]u_{t-1}^2 + \beta^* \sigma_{t-1}^2, \tag{2}$$

where $\mathcal{I}$ denotes an indicator function. For simplicity, we take y_0 and y_{-1} as fixed, and let $u_0 = y_0 - \phi_1 - \phi_2 y_{-1}$, $\sigma_0^2 = y_0^2$.

The density of u_t has a complicated analytical form which involves a Bessel function of u_t; see Aas and Haff (2006, Eq. (8)). Evaluating the Bessel function is very time-consuming. Fortunately, u_t can be shown to have the following mixture representation:

$$u_t = \sigma_t \left(\beta \left[Z_t - \frac{\delta^2}{\nu - 2} \right] + \sqrt{Z_t} \epsilon_t \right), \tag{3}$$

where ϵ_t is a standard Normal random variable, and where Z_t is independent of ϵ_t and has the following inverted Gamma density:

$$f(Z_t) = \frac{\left(\frac{\delta^2}{2}\right)^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)} Z_t^{-\nu/2-1} \exp\left[-\frac{\delta^2}{2Z_t} \right], \tag{4}$$

with

$$\delta^2 = \frac{(\nu-2)(\nu-4)}{4\beta^2} \left(-1 + \sqrt{1 + \frac{8\beta^2}{\nu - 4}} \right). \tag{5}$$