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Localized level crossing random walk test robust to the presence of structural breaks[☆]

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ABSTRACT

A modified version of the nonparametric level crossing random walk test is proposed, in which the crossing level is determined locally. This modification results in a test that is robust to unknown multiple structural breaks in the level and slope of the trend function under both the null and alternative hypotheses. No knowledge regarding the number or timing of the breaks is required. An algorithm is proposed to select the degree of localization in order to maximize bootstrapped power in a proximate model. A computational procedure is then developed to adjust the critical values for the effect of this selection procedure by replicating it under the null hypothesis. The test is applied to Canadian nominal inflation and nominal interest rate series with implications for the Fisher hypothesis.

1. Introduction

Originally examined by Kendall (1953), the random walk hypothesis has been important in numerous disciplines including economics, finance, and international finance, in which random walk models have been used to model variables such as consumption (Hall, 1978; Molana, 1991), stock prices (Mandelbrot, 1964; Samuelson, 1965; Black and Scholes, 1973), and exchange rates (Meese and Rogoff, 1983; Engel and West, 2005). Likewise, tests of the random walk hypothesis are frequently employed to test deeper theoretical models, such as the permanent income hypothesis (Hall, 1978) and weak form market efficiency (e.g. Lo and MacKinlay (1988)). Tests of weak form market efficiency include serial correlation tests, runs tests, and multiple variance ratio tests.

It is typical in such tests to allow for a linear trend. This is often necessary on economic grounds as well. For example, economic growth and inflation give rise to an upward long-run trend in stock prices. On the other hand, few of the tests mentioned above allow for changes or breaks to occur in the trend term. This is arguably a somewhat restrictive assumption, which may result in unreliable inference. For example, changes to the trend growth rates or long-term average inflation rates would imply a break in trend for nominal stock prices.

An alternative perspective on the random walk test is to view it as a special case of a unit root test with uncorrelated errors. In the unit root testing literature there has been a long and ongoing interest in robustifying inference to the presence of structural breaks. Perron (1989) and Perron and Vogelsang (1992) demonstrate that structural breaks can cause difficulty for unit root tests, by causing an I(0) series with a break to resemble an I(1) process near the break point. Perron (1989)

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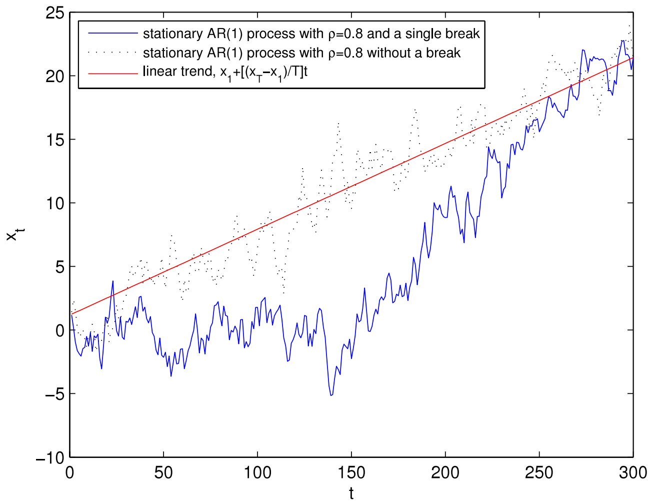

Fig. 1. The dotted black line shows a simulated stationary series with a linear trend. The solid blue line shows a stationary series with a break in the slope of the trend in the middle of the data (at $t = 150$, with a magnitude of 0.1). The straight solid red line shows the estimated trend line for the series with the break using the approach of Garcia and Sanso (2006).

proposed the first unit root test that allows for the possibility of a break. His model allows for a break under both the null and alternative hypotheses, but with exogenously determined break dates. The next generation of tests, for example Zivot and Andrews (1992) and Banerjee et al. (1992), allows for endogenously determined break dates under the alternative (but not the null) hypothesis. Most recently, Kim and Perron (2009) extend the earlier test of Perron (1994) to allow for a break in a trend function at an unknown time under both the null and alternative hypotheses.

A second issue that complicates unit root testing in the presence of structural breaks is the risk of misspecification with respect to the number of structural breaks. Vogelsang (1994) shows that the power of a unit root test is non-monotonic when a one-break model is estimated on data that contain two breaks. Lumsdaine and Papell (1997) consider testing the unit root hypothesis, while allowing for two endogenous breaks under the alternative hypothesis. Ohara (1999) develops a unit root test that allows for an alternative hypothesis with multiple trend breaks at unknown dates. Kapetanios (2005) provides tests for the unit root hypothesis against the occurrence of an unknown, but finite number of breaks.

In this paper we propose a level crossing random walk test that is robust to the possible presence of multiple structural breaks. This approach builds on a rich literature involving level crossings and their application to random walk and unit root testing. Studies on level crossings by continuous stationary Gaussian processes used in models of physical phenomena date back at least sixty years. Discretization of the continuous schemes and its applications to economics and finance begins with the works of Ho and Sun (1987), Burridge and Guerre (1996), and Ho and Hsing (1997) to name a few.

Interestingly, in one of the earliest examinations of the random walk (without drift) hypothesis, Cowles and Jones (1937) compared the frequency of sequences and reversals in historical stock returns. This could be interpreted as an informal level crossing test using a crossing level of zero.

Burridge and Guerre (1996) proposed a formal nonparametric unit root test based on the standardized number of level crossings of a random walk without deterministic terms (drift or trend). They observed that in the presence of a unit root, a time series, x_t, would only infrequently cross any crossing level μ. They therefore used the empirical frequency of this event to distinguish between random walks and stationary processes. They showed that the asymptotic distribution of the test statistic relates to a scaled local time of a Brownian motion and found that the scale factor depends on the long-run variance of the process.

Garcia and Sanso (2006) extend the work of Burridge and Guerre (1996) in two aspects. First, they allow for more general structures of autocorrelated disturbances. Secondly, they allow for a linear time trend and propose a modified crossing statistic, based on the frequency with which x_t crosses the time trend, which they estimate by $x_1 + (\frac{x_T - x_1}{T})t$, i.e. by fitting a line through the two sample endpoints. Thus, instead of using a fixed crossing level, their crossing level $\hat{\mu}_t$ follows the (estimated) linear trend, i.e. $\hat{\mu}_t = x_1 + (\frac{x_T - x_1}{T})t$. Equivalently, their procedure is based on the frequency with which the linearly detrended series, $x_t - x_1 - (\frac{x_T - x_1}{T})t$, crosses zero.

Although Garcia and Sanso (2006) allow for a linear trend, neither they nor Burridge and Guerre (1996) allow for the possibility of trend breaks. While these methods work quite well in the no break case for which they were designed, the crossing statistics constructed in these ways are susceptible to structural breaks. In Fig. 1 the number of crossings, on which the Garcia and Sanso (2006) test statistic relies, will be equal to the number of crossings of the straight trend line and the stochastic process. It is apparent that for a stationary process with a single break in the middle, as depicted in the figure, the number of crossings will be minimal. The larger the magnitude of the break, the larger will be the loss in power. The tests are particularly sensitive to breaks occurring towards the middle of the sample.