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Structural model of credit migration

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ABSTRACT

Credit migrations constitute the building blocks of modern risk management. A firm-specific structural model of credit migration that incorporates the firm's capital structure and the risk perception of rating agencies is proposed. The proposed model employs the notion of distance-to-default, which quantifies default probability. The properties of Brownian excursions play an essential role in the analysis. The proposed model not only allows the derivation of closed-form credit transition probability, but also provides plausible explanations for certain empirical evidence, such as the default probability overlaps in ratings and the slow-to-respond feature of rating agencies. The proposed model is calibrated through simulations and applied to empirical data, which show rating agencies' risk perceptions to be significant. The calibrated model allows calculation of the firm-specific transition probabilities of rated companies.

1. Introduction

Credit risk management is an important topic in finance, especially in the high-yield bond and bank loan markets. Of critical concern to financial firms are changes in the credit ratings of the products in their portfolios. Nationally recognized statistical rating organizations (NRSROs), such as Standard & Poor's Rating Services, Moody's Investor Services and Fitch Ratings, assign corporate bond issuers different credit ratings to reflect their creditworthiness. Credit risk managers pay particular attention to the ratings and transition matrices published by these NRSROs.

Most of the research in this arena makes use of historical transition matrices and firm ratings to estimate future transition probabilities; see, for example, Altman and Kao (1992), Kavvathas (2000), and Lando and Skodeberg (2002). There are many ways to handle credit migration, for example, the simulation-based Bayesian approach (Stefanescu et al., 2009), the factor probit model (Feng et al., 2008), the credit barrier model (Albanese and Chen, 2006), the random effects multinomial regression model (Kim and Sohn, 2008) and the Markov mixture models (Frydman and Schuermann, 2008). Fuertes and Kalotychou (2006) consider the role of heterogeneity in early warning systems for sovereign debt crises. In a later study, they (Fuertes and Kalotychou, 2007) investigated the estimations and rating dynamics of sovereign credit migrations.

Few studies consider the slow-to-respond feature of rating agencies in the process of credit rating changes. Although the possible conflict of interest between NRSROs and credit derivative issuers was of great concern during the sub-prime crisis of 2008–2009, a summary report issued by the US Securities and Exchange Commission (Luca and Russo, 2009) found rating agencies' surveillance process slow to respond to the up-to-date credit situation. In fact, Kealhofer et al. (1998) had already demonstrated the significant overlap that exists in the expected default probabilities (EDPs) across letter grades, pointing out that this overlap may arise from the slow-to-respond feature of rating agencies.

Here, we model this feature by excursion time, which measures the consecutive time that a firm's credit index lies outside its credit rating zone. More specifically, we employ the notion of distance-to-default, which is based on a firm's

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Table 1
Averaged one-year transition matrix (1984–2000) from Moody's.

% Initial rating (k[a])	Terminal rating							
	Default	Caa/C	B	Ba	Baa	A	Aa	Aaa
Caa/C(1)	16.38	74.2	6.46	2.02	0.94	0.00	0.00	0.00
B(2)	7.79	2.63	83.09	5.56	0.67	0.17	0.07	0.02
Ba(3)	1.72	0.33	9.32	83.11	4.90	0.56	0.03	0.02
Baa(4)	0.08	0.07	0.87	4.73	89.30	4.85	0.09	0.00
A(5)	0.00	0.00	0.21	0.79	5.49	91.64	1.84	0.04
Aa(6)	0.00	0.00	0.04	0.08	0.56	9.86	88.78	0.69
Aaa(7)	0.00	0.00	0.00	0.00	0.00	0.37	7.10	92.52

[a] Each letter grade is assigned an integer k.

market and book values, such as the credit index. Drawing on Gordy and Heitfield (2001), we map the distance-to-default into different rating categories by partitioning the data. Following financial market convention, the distance-to-default dynamic is assumed to follow a geometric Brownian motion. The slow-to-respond feature is then described by the Brownian excursion time that the distance-to-default value falls outside the range of a firm's credit rating region. This Brownian excursion time is referred to as migration duration in this paper. The proposed model allows for the overlap of EDPs across different letter grades, and offers a means to reconcile the empirical findings of Kealhofer et al. (1998). A closed-form formula for migration probability is obtained to estimate and calibrate the model parameters by means of computational statistics approaches. In essence, our approach can be considered as a reverse engineering exercise that allows features such as slowness to respond to be incorporated.

The use of distance-to-default to model credit risk is commonly known as the structural default model approach in the finance literature. There are several reasons to adopt this structural approach. First, it is conceptually sound and easy to implement by means of suitable statistical techniques; see Duan (1994), Duan et al. (2004), and Wong and Choi (2009). Second, there is empirical evidence to show that the combined use of structural models and statistical estimation provides accurate predictions of corporate bonds with different credit ratings; see Ericsson and Reneby (2005) and Li and Wong (2008). Third, the distance-to-default concept takes three key credit issues into account: the value of the firm's assets, its business and industry risk, and its leverage. Finally, rating agencies themselves have begun using structural approaches to calculate the EDPs of firms.

The remainder of the paper is organized as follows. Section 2 introduces the empirical transition probabilities and the distance-to-default measure. The proposed model is formulated in Section 3, in which the analytical formulas of the transition probabilities are also derived. Section 4 demonstrates the calibration and estimation procedure through a simulation, and then reports the results and discusses the implications of the empirical data analysis. Concluding remarks are given in Section 5.

2. Credit transition and distance-to-default

2.1. Empirical transition probabilities

Let $C_k, k = 0, 1, 2, \dots, n$ be $n+1$ credit classes arranged in ascending order. Thus, C_0 stands for the default grade, and C_n designates the best credit class. The empirical transition probabilities published by rating agencies employ a discrete-time setting and rely on a cohort method. Suppose that N_i firms belong to category C_i at the beginning of the year, out of which N_{ij} migrate to category C_j by the end of the year. The one-year transition rate is then estimated as

$$\hat{p}_{ij} = \frac{N_{ij}}{N_i}, \quad i, j = 0, 1, \dots, n. \tag{1}$$

The history of rating transition data is summarized into rating transition matrices. Table 1 shows the yearly averaged transition matrix (for the years 1984–2000) released by Moody's Investment Services.

A number of interesting features in this empirical transition matrix are worth noting. First, the overwhelming majority of firms remain in the same rating category throughout the year. Second, firms are more likely to be downgraded than upgraded. Third, no transitions take place in certain categories, although the corresponding true transition probabilities may not necessarily be zero. Finally, the average default frequencies rise dramatically as one moves from the higher to the lower credit ratings. It will be demonstrated here that these stylized features can be captured by the proposed model, but not by traditional structural models.

Consider the transition probability that a representative C_k-firm migrates to C_j for all $j \in \mathcal{J}$, an index set. This probability can be estimated by

$$\hat{p}_{k \to \mathcal{J}} = \sum_{j \in \mathcal{J}} \hat{p}_{kj},$$