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Specification tests for the error distribution in GARCH models

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ABSTRACT

Goodness-of-fit and symmetry tests are proposed for the innovation distribution in generalized autoregressive conditionally heteroscedastic models. The tests utilize an integrated distance involving the empirical characteristic function (or the empirical Laplace transform) computed from properly standardized observations. A bootstrap version of the tests serves the purpose of studying the small sample behaviour of the proclaimed procedures in comparison with more classical approaches. Finally, all tests are applied to some financial data sets.

1. Introduction

Suppose that a process is driven by a generalized autoregressive conditionally heteroscedastic (GARCH) model of specific order. This class of models was introduced by Bollerslev (1986), and despite the fact that certain properties of GARCH processes make no use of the particular form of the innovation distribution (see, for example, the stationarity conditions mentioned later in this section), GARCH models with Gaussian innovations have been considered. However, on the basis of numerous real-data applications accumulated over the years, the normality assumption has become questionable, not only for the marginal, but also for the conditional distribution of certain financial quantities. Specifically, it often appears that such quantities are skewed and contain a persistent amount of leptokurtosis; refer, for instance, to Bollerslev (1987), Nelson (1991), Ding (1995), Curto et al. (2009), Mittnik and Paolella (2003) and Mittnik et al. (1998).

Although maximum-likelihood-type estimators for GARCH models are consistent and asymptotically normal under mild conditions, regardless of the error distribution, the impact of an incorrect Gaussian assumption has been investigated by several researchers. Engle and González-Rivera (1991) show that there is a loss of efficiency as high as 84% of the so-called quasi-MLE (QMLE) of the parameters under non-normal innovations. González-Rivera and Drost (1999) theoretically show that the efficiency of the QMLE and a certain semiparametric estimator, relative to the MLE, depends on the kurtosis as well as the skewness of the conditional error density. In fact, and unlike the case of symmetry which allows for equal efficiency of the three estimators, the presence of skewness rules out the possibility of a fully efficient QML estimator. On the other hand, a QMLE associated with an incorrect non-Gaussian specification may also imply inconsistency. Specifically, Newey and Steigerwald (1997) show that in certain conditional autoregressive models nesting the GARCH, inconsistency may result not from the presence of non-Gaussian errors alone, but if either the assumed or the true error density is asymmetric. For simulation results on the behaviour of the QML estimator under misspecified GARCH(1, 1) models, see Bellini and Bottolo (2009). Further evidence from Huang et al. (2008) suggests that least absolute deviations estimators may be preferred to

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maximum likelihood estimation under Laplace and certain Student's t-innovation distributions. There is also considerable evidence on the impact of correct specification on other aspects of modelling, such as predictions; see, for instance, Hansen and Lunde (2005), Forsberg (2002) and Bellini and Bottolo (2009).

Despite the aforementioned importance of the innovation distribution, the literature contains few references to corresponding specification tests. Kulperger and Yu (2005), for example, modify the well-known test of Jarque and Bera (1987) (based on skewness and kurtosis) with respect to the case of GARCH specification in order to test the normality of the innovation distribution. Horváth and Zitikis (2006) propose a general goodness-of-fit test based on a nonparametric estimator of the innovation density and Horváth et al. (2004) suggest certain modifications of classical statistics based on the distribution function of squared innovations, while Koul and Ling (2006) propose a weighted version of the Kolmogorov–Smirnov test which has a model-free asymptotic null distribution.

In this paper, we propose goodness-of-fit tests for the innovation distribution based on the 'Fourier approach'. This approach utilizes the characteristic function in order to test the corresponding null hypothesis. Specifically, we consider test statistics for GARCH(p, q) models which are based on the empirical characteristic function (ECF)

$$\hat{\varphi}_T(u) = \frac{1}{T-m}\sum_{t=m+1}^{T} \mathrm{e}^{\mathrm{i}u\hat{\varepsilon}_t}, \quad -\infty < u < \infty,\ \hat{\varepsilon}_t = y_t/\hat{c}_t, \tag{1.1}$$

where $m = \max(p, q)$. The corresponding observations $y_1, \dots, y_T$ are from the GARCH model of fixed order (p, q)

$$y_t = c_t\varepsilon_t, \qquad c_t^2 = \beta_0 + \sum_{j=1}^{q}\beta_j y_{t-j}^2 + \sum_{j=1}^{p}\gamma_j c_{t-j}^2,$$
$$\beta_0 > 0, \qquad \beta_j \ge 0 \quad (1 \le j \le q), \qquad \gamma_j \ge 0 \quad (1 \le j \le p),$$

incorporating the i.i.d. innovations $\varepsilon_t,\ t = 1, \dots, T$, with the empirical scales $\hat{c}_t$ being computed from an estimate of the unknown parameter-vector $\vartheta = (\beta_0, \dots, \beta_q, \gamma_1, \dots, \gamma_p)$. The GARCH$(p, q)$ model is covariance stationary if and only if $\sum_{j=1}^{q}\beta_j + \sum_{j=1}^{p}\gamma_j < 1$. Strict stationarity conditions have also been derived by Bougerol and Picard (1992a,b). Considering the GARCH$(1, 1)$ model, for example, the necessary and sufficient condition reads as $\mathbf{E}[\log(\gamma_1 + \beta_1\varepsilon^2)] < 0$.

The remainder of the paper is structured as follows. In Section 2, we introduce the tests and discuss some aspects of the test statistics, as well as the important issue of estimation of parameters. Bootstrap versions of the tests are introduced in Section 3 and their behaviour is studied by means of Monte Carlo techniques. In Section 4, we apply the methods to some real data, and finally our findings are summarized in Section 5.

2. Test statistics and estimation

(i) *Test statistics*. The test statistics based on the ECF are analogous to the corresponding goodness-of-fit tests for i.i.d. observations. In particular, $F(\cdot)$ and $\varphi(\cdot)$ denote the distribution function and the characteristic function of the innovations, respectively. Despite the fact that only tests for specific distributions are considered in the following, we formulate the null hypothesis as $H_0 : F \in \mathcal{F}$, where $\mathcal{F}$ denotes a parametric family of distributions. Thus, the hypothesized distribution is allowed to depend on an unknown parameter, which ought to be estimated from the data; we shall come back to this issue in Section 5. The test statistic then takes the form

$$\Phi_{T,w} = (T-m)\int_{-\infty}^{\infty} |\hat{\varphi}_T(u) - \varphi(u)|^2 w(u)\mathrm{d}u, \tag{2.1}$$

with $w(u)$ denoting an appropriate weight function introduced in order to taper the persistent periodic behaviour of $\hat{\varphi}_T(u)$. Note that in the context of i.i.d. observations, the limit behaviour of the ECF was systematically studied in a series of papers by Feuerverger and Mureika (1977), Csörgő (1981), Marcus (1981), and Csörgő and Totik (1983). The amiable limit properties of the ECF have furthered the use of L2-test statistics in the style of Eq. (2.1). In turn, the theoretical properties of these statistics have been investigated by Henze and Wagner (1997), Meintanis (2004), Epps (2005) and Matsui and Takemura (2007) for goodness-of-fit tests with some particular cases of parametric distributions, and by Meintanis and Swanepoel (2007) under a general formulation in a Hilbert space setting, including assumptions on the weight function $w(u)$ and the estimators of parameters involved. However, concerning the actual performance of the tests, the limit null distribution derived is extremely difficult to employ, and we do not pursue this issue any further at this point. On the other hand, these earlier studies have shown that the ECF statistics can be more powerful and/or more convenient to use than the more standard procedures.

To gain some insight into the method proposed, suppose that $w(-u) = w(u)$ and write $C(u)$ (resp. $S(u)$) for the real part (resp. imaginary part) of the characteristic function of the innovations $\varphi(u)$. The test statistic then may be written as

$$\Phi_{T,w} = (T-m)\int_{0}^{\infty} g(u)w(u)\mathrm{d}u, \tag{2.2}$$