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Bayesian inference in a Stochastic Volatility Nelson–Siegel model

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ABSTRACT

Bayesian inference is developed and applied for an extended Nelson–Siegel term structure model capturing interest rate risk. The so-called Stochastic Volatility Nelson–Siegel (SVNS) model allows for stochastic volatility in the underlying yield factors. A Markov chain Monte Carlo (MCMC) algorithm is proposed to efficiently estimate the SVNS model using simulation-based inference. The SVNS model is applied to monthly US zero-coupon yields. Significant evidence for time-varying volatility in the yield factors is found. The inclusion of stochastic volatility improves the model's goodness-of-fit and clearly reduces the forecasting uncertainty, particularly in low-volatility periods. The proposed approach is shown to work efficiently and is easily adapted to alternative specifications of dynamic factor models revealing (multivariate) stochastic volatility.

1. Introduction

Modeling the term structure of interest rates is of importance in many areas in financial economics and macroeconomics. In finance, information revealed by the yield curve is important for the pricing of bonds and interest rate derivatives, as well as for portfolio management and asset allocation. In macroeconomics, the yield curve carries important information about the state of the economy and business cycles. Whereas traditional approaches such as those of Vasicek (1977), Cox et al. (1985), or Hull and White (1990) focus on equilibrium or no-arbitrage relationships, interest rate dynamics are typically captured in terms of factor models such as the model proposed by Nelson and Siegel (1987). Diebold and Li (2006) reformulate the Nelson–Siegel model – henceforth referred to as the DLNS model – in terms of a state–space representation. The DLNS model allows for a two-step estimation of the factors and dynamics in the factor loadings. Koopman et al. (2010) and Hautsch and Ou (2008) extend the Nelson–Siegel model to allow for time-varying volatility. Whereas Koopman et al. (2010) allow for a common volatility component in all yield processes, Hautsch and Ou (2008) propose capturing stochastic volatility in the underlying yield factors associated with level volatility, slope volatility and curvature volatility. Whereas the Stochastic Volatility Nelson–Siegel (SVNS) model, proposed in Hautsch and Ou (2008), is a powerful approach to parsimoniously capture dynamics in yields and corresponding volatilities, the statistical inference of such a model is not straightforward because both yield factors and volatility factors are unobservable.

A main contribution is to develop Bayesian inference for this class of models. An efficient algorithm is suggested that allows the highly correlated latent variables to be drawn in only a few blocks. The importance of accounting for stochastic volatility in the Nelson–Siegel model and how to extract the unobservable volatility components from the data are illustrated. Though the estimation procedure is specifically designed for the SVNS model, it is easily adapted to alternative factor specifications. In this sense, the proposed algorithm provides a general framework for the estimation of dynamic factor

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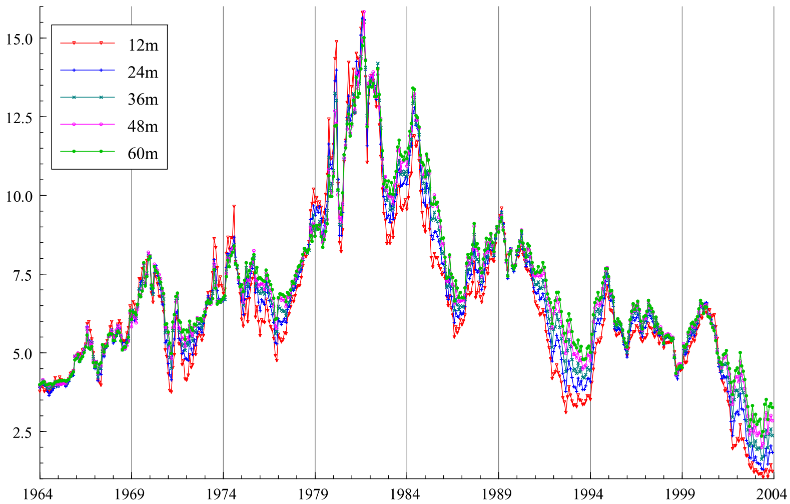

Fig. 1. Time series plots of US zero-coupon yields, Jan. 1964–Dec. 2003. Maturities: 12, 24, 36, 48, 60 months, respectively.

models revealing (multivariate) stochastic volatility. In an empirical application to US bond yields, the proposed procedure works well and allows us to efficiently extract unobservable time-varying volatility components.

The exponential components factor model proposed by Nelson and Siegel (1987) acts as a workhorse for the estimation and prediction of yield curves and is extensively used in financial practice and central banks. Its power mainly stems from the fact that it is easy to implement and sufficiently flexible to capture a wide range of possible shapes of the yield curve. Though it is neither an equilibrium nor a no-arbitrage model, many banks use this framework to construct zero-coupon yield curves. Various extensions of the Nelson–Siegel model have been proposed, see, e.g., Björk and Christensen (1999), Rudebusch and Wu (2008), Diebold et al. (2005) and Diebold et al. (2006). Diebold and Li (2006) propose a dynamic version of the Nelson–Siegel model by allowing the underlying factor loadings to vary over time following a vector autoregressive (VAR) structure. As shown by Diebold and Li (2006) and Diebold et al. (2006), the model is able to capture interest rate dynamics and to successfully predict future yield curves. These contributions provide a new way to model interest rate dynamics using factor models and complement the class of no-arbitrage affine models (Vasicek (1977), Cox et al. (1985), Duffee and Kan (1996), Dai and Singleton (2000) or Sanford and Martin (2005), among others). See Piazzesi (2003) for a survey on affine models in continuous time.

Fig. 1 depicts time series plots of yields with different maturities stemming from the data underlying this study. It can be observed that yields with different maturities are closely related and the yield trends are similar. Further evidence for time-varying volatility in the interest rate series is found. Particularly in the 1980s, yields for all maturities were very volatile. However, capturing time-varying volatility in yield curves is challenging due to their high dimensionality. Koopman et al. (2010) extend the dynamic Nelson–Siegel model by allowing for a common volatility component jointly affecting the yield processes for all maturities. A common volatility component can be associated with the volatility of an underlying bond market portfolio in the spirit of Engle et al. (1990). However, such a specification is not flexible enough to capture specific maturity-dependent volatilities. As a more flexible but still parsimonious alternative, Hautsch and Ou (2008) propose directly modeling stochastic volatility in the yield curve factors. In this case, the time-varying volatilities in individual yields are captured by yield factor volatilities. These volatilities are naturally interpreted as the volatilities of underlying bond portfolios associated with short-term, medium-term and long-term maturities.

However, extracting the latent factors as well as their time-varying volatility components is challenging. An efficient MCMC algorithm to conduct Bayesian inference in the SVNS model is proposed that contributes to the literature on the estimation of (extended) multivariate dynamic factor models. To extract the factors, the Kalman filter algorithm embedded in an MCMC procedure is suggested. The unobservable time-varying volatilities are extracted using an approximating re-weighting approach proposed by Kim et al. (1998) and Chib et al. (2002). Using the suggested MCMC algorithm, all latent yield factors, stochastic volatilities and parameters can be sampled at once in a few blocks. MCMC diagnostics results show that the proposed procedure is computationally quite efficient and is clearly superior to an element-by-element sampling of the underlying parameters and latent factors as used by e.g., Hautsch and Ou (2008). The latter procedure requires an enormous number of Monte Carlo drawings making the model intractable if the sample size is high.

Applied to monthly zero-coupon yields covering a period from 1964 to 2003, the SVNS model is able to successfully capture the dynamics in time-varying yields and volatilities thereof. There is strong evidence for time-varying volatility in interest rates. All yield volatilities reveal a high persistence. To explain the volatilities in the underlying yield curve, the level and slope volatility are particularly important. To evaluate the model's goodness-of-fit, posterior predictive p-values are computed. Results show that the explicit inclusion of stochastic volatilities in level, slope and curvature factors improves