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Resistance distance in complete n-partite graphs

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ABSTRACT

We may view any graph as a network of resistors each having a resistance of 1 Ω. The *resistance distance* between a pair of vertices in a graph is defined as the effective resistance between the two vertices. This function is known to be a metric on the vertex-set of any graph. The main result of this paper is an explicit expression for the resistance distance between any pair of vertices in the complete n-partite graph $K_{m_1,m_2,\ldots,m_n}$.

1. Introduction

We shall deal here with finite graphs without loops but possibly with multiple edges. The vertex-set of a graph G is denoted by $V(G)$ and its edge-set is denoted by $E(G)$.

The usual *distance* from a vertex x to a vertex y in a graph G is defined to be the length of any shortest path joining x to y. Thus $d\colon V(G) \to \mathbb{R}$ is a function that satisfies

(1) $d(x, y) \geq 0$ for all $x, y \in V(G)$,
(2) $d(x, y) = 0$ if and only if $x = y$,
(3) $d(x, y) = d(y, x)$ for all $x, y \in V(G)$,
(4) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in V(G)$.

If A is any set, any function μ from $A \times A$ to the set $\mathbb{R}$ of real numbers satisfying the four conditions above is called a *metric* on A.

Here we shall deal with a certain metric on the vertex-set of a graph G called *resistance distance*.

We associate a graph G with a network $N(G)$ of unit resistors (resistor with resistance 1 Ω) in the most natural way—each edge of G is a unit resistor in $N(G)$. For example, the fan F_3 and the associated network $N(F_3)$ are shown in Fig. 1.

If a source of electromotive force is connected to two nodes of the network, say at 0 and 1, current will flow into and out of the network. According to Ohm's law, if the difference in potential between two nodes is V and the current that flows into one node and out of another node is I, then $V = IR$, where R is the effective resistance between the two nodes. Please refer to Fig. 2.

Ohm's law easily yields formulas for the effective resistance of *resistors in series* or *resistors in parallel*.

Fig. 3 shows n resistors connected in series.

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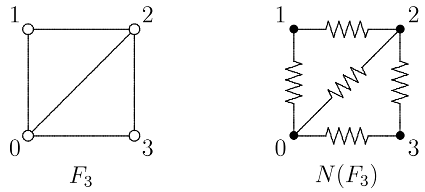

Fig. 1. The fan F_3 and the associated network of unit resistors $N(F_3)$.

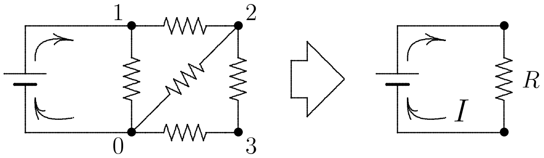

Fig. 2. An electrical circuit with effective resistance R between 0 and 1.

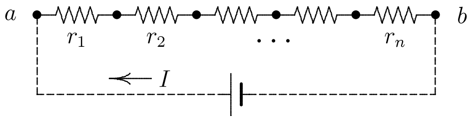

Fig. 3. Resistors in series with respect to a and b.

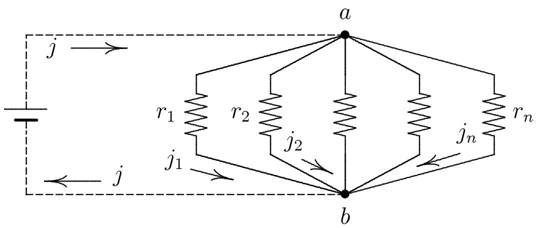

Fig. 4. Resistors in parallel with respect to a and b.

According to Ohm's law, the current that flows through a resistor in an electrical circuit is equal to the potential difference between the terminals of the resistor divided by the resistance of the resistor. If V is the potential difference, i is the current, and R is the resistance, then $i = V/R$. Here, resistor may be replaced by a network of resistors.

If n resistors are in series, the current that flows through each of them is the same. Referring to Fig. 3, let the potential at a be V_a and that at b be V_b. Then the potential difference between a and b is $V = V_a - V_b$ if we assume that the potential at a is higher. Potential is much like pressure. We can compare the situation to water flowing in a pipe through loads $r_1, \ldots, r_n$. There are different pressures at points between adjacent loads. The summation of all pressure differences from a to b is equal to the difference between the pressures at a and b. Going back to the original set-up of resistors, the potential difference between the terminals of r_1 is ir_1. The summation of all potential differences is

$$i(r_1 + r_2 + \cdots + r_n).$$

If we view the resistors in series as one single resistor with resistance R, then the potential difference between a and b is iR. Therefore, we get the equation

$$iR = i(r_1 + r_2 + \cdots + r_n).$$

Hence, if n resistors with resistances of $r_1, r_2, \ldots, r_n$ ohms are in series, their effective resistance is

$$\Omega(a, b) = r_1 + r_2 + \cdots + r_n. \tag{1}$$

Fig. 4 shows n resistors in parallel. Assume that the total current entering at a is j and that the current flowing through r_i is j_i. Then $j = j_1 + j_2 + \cdots + j_n$.

If we denote by R the effective resistance between a and b, then $j = V/R$, where V is the potential difference between a and b. But a and b are common terminals of all the resistors. Therefore, we have

$$j = j_1 + j_2 + \cdots + j_n$$
$$\frac{V}{R} = \frac{V}{r_1} + \frac{V}{r_2} + \cdots + \frac{V}{r_n}.$$