Discrete Applied Mathematics 201 (2016) 141–149

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Conditional diagnosability of bubble-sort star graphs[☆]

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ARTICLE INFO

Article history:
Received 20 December 2014
Received in revised form 2 May 2015
Accepted 24 July 2015
Available online 15 August 2015

Keywords:
Interconnection networks
Conditional diagnosability
MM model
PMC model
Bubble-sort star graphs

ABSTRACT

Diagnosability plays an important role in measuring the reliability of interconnection networks. Conditional faulty set is a special faulty set that does not contain all of neighbors of any vertex in a network. The conditional diagnosability is a metric that can give the maximum cardinality of the conditional faulty sets that the system is guaranteed to identify. This paper shows that the conditional diagnosability of the bubble-sort star graph BS_n under the MM model is $6n-15$ for $n \geq 6$ and under the PMC model is $8n-21$ for $n \geq 5$.

1. Introduction

With the rapid development of technology, a multiprocessor system may contain thousands of processors. As a significant increase in the number of processors, fault diagnosis of interconnection networks has become increasingly important. *Diagnosis* of a system is a process of identifying faulty nodes with fault-free nodes. The maximum number of faulty nodes that a system is guaranteed to identify is called the *diagnosability* of the system. The diagnosability of many interconnection networks has been explored. For example, see [5,8–10,14,16–24].

For the purpose of self-diagnosis of a system, some different models have been proposed. Among the proposed models, the MM model in [17] and the PMC model in [18] are widely used. The MM model is also called *comparison diagnosis model*, in which, the diagnosis is performed by sending the same input from a processor (node) to each pair of its distinct neighbors, and then comparing their outcomes. Sengupta and Dahbura [19] suggested the MM* model which is a modification of the MM model. In the MM* model, every processor must test another two processors if it is adjacent to them. Only a fault-free processor can guarantee reliable outcome, and the output are identical if the two processors which are adjacent to it are fault-free and distinct otherwise. The MM model was adopted in [7,20]. In the PMC model, every processor can test the processor that is adjacent to it and only the fault-free processor can guarantee reliable outcome. The PMC model was adopted in [2,14,22].

Given a system, it is not possible to determine whether some processor u is fault-free or not, if all the neighbors of processor u are faulty. In this case, Lai et al. [14] proposed the *conditional faulty set*, which is a special faulty set that does not contain all of neighbors of any vertex in a network. The *conditional diagnosability* is a metric that can give the maximum number of conditional faulty set that the system is guaranteed to identify. Many researchers have studied the conditional diagnosability of different networks under different models. The conditional diagnosability of a k-Ary n-Cubes Q_n^k is $6n-5$ for $n \geq 4$ and $k \geq 4$ under the MM model [11] and $8n-7$ for $n \geq 4$ and $k \geq 4$ under the PMC model [4]. The conditional

☆ This work is partially supported by NNSFC (No. 61373019, 11171097).
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http://dx.doi.org/10.1016/j.dam.2015.07.026
0166-218X/

diagnosability of the n-dimensional hypercube Q_n is $3n-5$ for $n \geq 5$ under the MM* model [13] and $4n-7$ under the PMC model [14]. The conditional diagnosability of folded hypercube FQ_n [12] is $3n-2$ for $n \geq 5$ under the MM* model. The conditional diagnosability of star graph S_n [21] is $n-1$ for $n \geq 4$ under the MM model and $8n-21$ for $n \geq 5$ under the PMC model [3]. The conditional diagnosability of Bubble-sort graph B_n [22] is $4n-11$ for $n \geq 4$ under the PMC model.

Clearly, the star graph owns many attractive properties except the embeddability as well as the Bubble-sort graph is simple and possesses some desirable features except the long diameter. So we may expect that the Bubble-sort star graph will combine the advantages of both graphs (see [1,6]). The remainder of this paper is organized as follows: Section 2 introduces some necessary definitions and notations. In Section 3, we demonstrate the conditional diagnosability of the Bubble-sort star graphs under the MM Model. In Section 4, we demonstrate the conditional diagnosability of the Bubble-sort star graphs under the PMC Model.

2. Preliminaries

The topology of an interconnection network is often modeled as an undirected graph, where vertex and edge represent the processor and the link between two processors, respectively. Throughout this paper, we only consider simple, undirected and connected graphs. Let $G=(V,E)$ be a graph with vertex set $V=\{v_1,v_2,\ldots,v_n\}$. For $v_i \in V$, the degree of v_i, written by d_i or $d(v_i)$, is the number of edges incident with v_i. Let $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degree of G, respectively. If $\Delta(G)=\delta(G)$, then the graph is *regular*. The set of neighbors of a vertex v_i in G is denoted by $N_G(v_i)$. For a subset S of V, $G[S]$ is a subgraph of G induced by S. The *neighborhood set* of S in G is defined as $N_G(S)=(\cup_{u\in S} N_G(u))-S$. We will use $G-S$ to denote the subgraph $G[V(G)-S]$. The minimum size of a vertex set $S \subseteq V(G)$ that the graph $G-S$ is disconnected or has only one vertex, denoted by $\kappa(G)$, is the *connectivity* of G. The *symmetric difference* of $F_1 \subseteq V(G)$ and $F_2 \subseteq V(G)$ is defined as the set $F_1 \triangle F_2=(F_1-F_2)\cup(F_2-F_1)$. Let $cn(G)$ be the maximum number of common neighbors between any two vertices in G.

In the MM model, a self-diagnosable system of a graph G is often represented by a multigraph $M(V,L)$, where V and L are the vertex set of G and the labeled edge set, respectively. $(u,v;w)$ is defined as a labeled edge, if vertices u and v are adjacent to w, which implies that u and v are being compared by w. Since a pair of vertices may be compared by different vertices, M is a multigraph. For $(u,v;w)\in L$, we use $\pi((u,v;w))$ to denote the result of comparing vertices u and v by w. For w being fault-free, if both u and v are fault-free, then $\pi((u,v;w))=1$; otherwise $\pi((u,v;w))=0$. If w is faulty, $\pi((u,v;w))$ may be either 1 or 0, which implies the result is unreliable. The collection of all comparison results in $M(V,L)$ defined as a function $\pi: L \to \{0,1\}$, is the *syndrome* of the diagnosis.

In the PMC model, a self-diagnosable system of a graph G is often represented by a digraph $D(V,L)$, where V and L are the vertex set of G and the order edge set, respectively. (u,v) is defined as an order edge, if vertex u is adjacent to v, which implies that u can test v. For $(u,v)\in L$, we use $\pi((u,v))$ to denote the result of testing vertex v by u. For u being fault-free, if v is fault-free, then $\pi((u,v))=1$; otherwise $\pi((u,v))=0$. If u is faulty, $\pi((u,v))$ may be either 1 or 0, which implies the result is unreliable. The collection of all comparison results in $D(V,L)$ defined as a function $\pi: L\to\{0,1\}$, is the *syndrome* of the diagnosis. This study assumes that each node u tests the other whenever they are adjacent to it.

A subset $F \subseteq V(G)$ is *compatible* with a syndrome π if the syndrome can arise the circumstance that all vertices in F are faulty while all vertices in $V(G)-F$ are fault-free. Since a faulty comparator w may return an unreliable result, a faulty set F may produce different syndromes. Let $\pi(F)$ be the set of all syndromes that is compatible with F. A system is said to be *diagnosable* if for every syndrome π, there is a unique $F\subseteq V(G)$ that is compatible with π. It is called *t-diagnosable* if the system is diagnosable as long as the size of faulty set does not exceed t. The maximum number of t that the graph G is t-diagnosable is called the *diagnosability* of G, written as $t(G)$.

A faulty set $F\subseteq V(G)$ is called a *conditional faulty set* if it does not contain all of neighbors of any vertex in G. Two distinct faulty subsets $F_1, F_2 \subseteq V(G)$ are *distinguishable* if $\pi(F_1)\cap\pi(F_2)=\emptyset$; otherwise, they are said to be *indistinguishable*. A system G is *conditional t-diagnosable* if every two distinct conditional faulty subsets $F_1,F_2\subseteq V(G)$ with $|F_1|,|F_2|\leq t$, are distinguishable. The *conditional diagnosability*, denoted by $t_c(G)$, is the maximum number of t such that the graph G is conditional t-diagnosable. Then $t_c(G)\geq t(G)$ [11].

Let $[a,b]=\{x: x$ is an integer with $a\leq x\leq b\}$, where a and b are integers. We denote "$\circ$" an operation such that $u=v\circ(i,j)$, for any $u=x_1x_2\cdots x_i\cdots x_j\cdots x_n$, $v=x_1x_2\cdots x_j\cdots x_i\cdots x_n$, where $x_i\in\{1,2,\ldots,n\}$ and $x_i\neq x_j$ ($i\neq j$, and $i,j\in[1,n]$). Now we give the definition of the bubble-sort star graph.

Definition 2.1 ([6]). The bubble-sort star graph BS_n has $n!$ vertices, each of which has the form $u=x_1x_2\cdots x_n$, where $x_i\in\{1,2,\ldots,n\}$ and $x_i\neq x_j$ for $i\neq j$, where $i,j\in[1,n]$. Any two vertices u and v of $V(BS_n)$ are adjacent if and only if $v=u\circ(1,i)$ for $i\in[2,n]$, or $v=u\circ(i-1,i)$ for $i\in[3,n]$.

Clearly, BS_n is $(2n-3)$-regular and vertex symmetry. Moreover, it is Hamiltonian and bipartite. We can partition BS_n into n subgraphs $BS_n^1, BS_n^2,\ldots,BS_n^n$, where every vertex $u=x_1x_2\cdots x_n\in V(BS_n^i)$ has a fixed integer i in the last position x_n for $i\in[1,n]$. For any vertex $u\in V(BS_n^i)$, we denote $u^+=u\circ(1,n)$, $u^-=u\circ(n-1,n)$, $N_u^+=\{u^+,u^-\}$. We let $E_{i,j}(BS_n)=E_{BS_n}(V(BS_n^i),V(BS_n^j))$ for simplicity, where $E_{BS_n}(V(BS_n^i),V(BS_n^j))$ denotes the edge set of BS_n with one end in $V(BS_n^i)$ and other end in $V(BS_n^j)$. It is obvious that BS_n^i is isomorphic to BS_{n-1} for $i\in[1,n]$. Fig. 1 illustrates BS_2, BS_3 and BS_4, respectively.