Discrete Applied Mathematics 196 (2015) 4–19

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Minimizing maximum weight of subsets of a maximum matching in a bipartite graph

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ARTICLE INFO

Article history:
Received 4 November 2013
Received in revised form 23 December 2014
Accepted 5 January 2015
Available online 2 February 2015

Keywords:
Bipartite graph
Maximal matching
NP-hardness in the strong sense
Non-approximability
Approximation algorithm

ABSTRACT

In a complete bipartite graph $G = (U, V, E)$ with weighted edges, set U of vertices is partitioned into disjoint subsets called components. The aim is to find an inclusion-wise maximal matching that minimizes the maximum weight of a component (sum of the weights of those edges incident to a vertex of the component). The well-known assignment problem is a special case of this problem when the number of components equals one. We prove that the problem is strongly NP-hard and that finding an approximate solution with the approximation ratio less than 2 is strongly NP-hard as well. We develop heuristics to find solutions that are less than one percent away from the optimum on average. The problem has been encountered while planning container transportation in rail-road transshipment yards.

1. Introduction

We consider the following optimization problem. There is a complete weighted bipartite graph $G = G(U, V, E)$ where U and V are the sets of vertices and E is the set of edges. Each edge $e = (u, v) \in E,\ u \in U,\ v \in V$, has a rational weight $w(e)$. Set U is partitioned into m disjoint subsets $U_1, U_2, \ldots, U_m$, the so-called components. For any maximal matching M, we define the weight of the component U_i as the sum of weights of the edges incident with a vertex belonging to this component: $w(U_i, M) = \sum_{u \in U_i, (u,v) \in M} w(u, v)$. The value of M is the maximum of the weights of the components: $w(M) = \max_{1 \le i \le m} w(U_i, M)$. The problem is to find a maximal matching of minimum value. We denote this problem by Min–Max Weighted Matching. See Fig. 1 as the illustration of the problem.

Note that we can concentrate on the case $|U| = |V|$ since the case $|U| \neq |V|$ reduces to this one by adding vertices with incident edges of zero weight.

The case when there is only one component in a partition (i.e., $U = U_1$, $m = 1$) is the well-known problem of finding a maximal matching of minimum weight in a complete bipartite graph that is equivalent to the assignment problem (see, e.g., Burkard et al. [4]). For the assignment problem there is a polynomial time algorithm of $O(n^3)$ complexity for its solution (see, e.g., Papadimitriou and Steiglitz [10]). The special case of the problem when $m = |U|$ is known as linear bottleneck assignment problem, see [5,6,8,9]. It is also polynomially solvable. In particular it can be solved by the threshold method in $O(n^2\sqrt{n/\log n})$ time (for details see, e.g., [4]). For arbitrary graphs and the case $m = 1$, Min–Max Weighted Matching is similar to the minimum weight edge dominating set problem which is NP-hard but admits a 2-approximation, see [7].

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http://dx.doi.org/10.1016/j.dam.2015.01.008

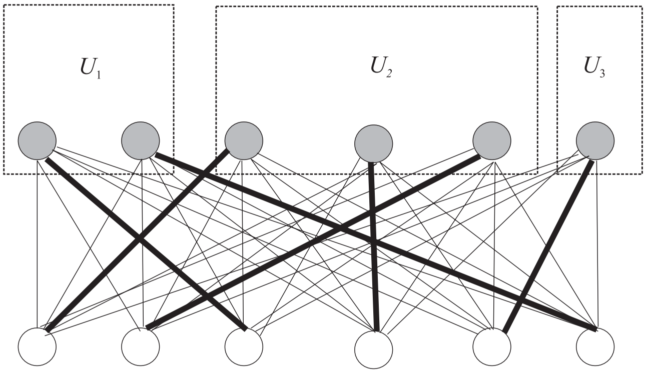

Fig. 1. The illustration of Min–Max Weighted Matching problem.

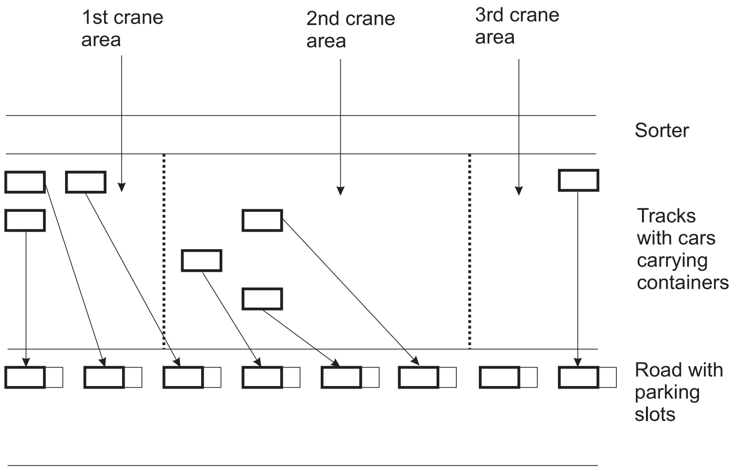

Fig. 2. The layout of the rail-road yard.

To the best of our knowledge, the Min–Max Weighted Matching problem with $1 < m < |U|$ has not been considered in the literature.

The problem is motivated by container transshipment in a rail-road terminal. In rail-road terminals, mainly serving as interface in intermodal transport, gantry cranes (see [2] for definition of and discussion on gantry cranes) transship containers between trains and trucks. A typical layout of a rail-road terminal is schematically depicted in Fig. 2. Freight trains are parked on parallel transhipment tracks of the terminal. Parallel to the train tracks is a road with a driving lane for trucks that flanks a lane with truck parking positions. In the most modern transshipment yards or MegaHubs, there is additionally a fully automated sorting system. Such a sorter consists of shuttle cars that can receive containers close to their initial positions from inbound trains and move them alongside the yard to their target positions. The goal is usually to minimize the transhipment time of all containers. For further details on a rail-road or transshipment yards structure, see Boysen et al. [2,3].

The situation within a rail-road terminal may be modeled in terms of the Min–Max Weighted Matching problem as follows. Set U is the set of all containers to be transshipped. Subset U_i is the set of all containers that are located in the ith crane area. Set V is the set of parking slots of the trucks. The positions of containers and parking slots are predefined and known. Moreover, the duration of transporting a specific container to a defined parking slot is known as well (loading and unloading time is taken into account while calculating these transportation times). These transportation times define the weights of the edges (they may be an approximation because a move from one area to another one will be performed through the sorter and we consider it as a direct single move by a crane). The problem is to assign each container to a parking slot so that the maximum sum of transportation times of the component will be minimal.

We take only moves of the cranes into account that carry containers—an assumption that proved to be realistic as mentioned in previous papers (see Boysen and Fliedner [1]).

In Section 2 we prove that the problem of finding an approximate solution for the Min–Max Weighted Matching problem with an approximation ratio less than 2 is strongly NP-hard. In Section 3 we present a basic approach for solving