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On the MAX MIN VERTEX COVER problem[☆]

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ABSTRACT

We address the MAX MIN VERTEX COVER problem, which is the maximization version of the well studied MIN INDEPENDENT DOMINATING SET problem, known to be **NP**-hard and highly inapproximable in polynomial time. We present tight approximation results for this problem on general graphs, namely a polynomial approximation algorithm which guarantees an $n^{-1/2}$ approximation ratio, while showing that unless $\mathbf{P} = \mathbf{NP}$, the problem is inapproximable within ratio $n^{\varepsilon-(1/2)}$ for any strictly positive ε. We also analyze the problem on various restricted classes of graphs, on which we show polynomiality or constant-approximability of the problem. Finally, we show that the problem is fixed-parameter tractable with respect to the size of an optimal solution, to treewidth and to the size of a maximum matching.

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1. Introduction

In the MIN INDEPENDENT DOMINATING SET problem, also called MIN MAX INDEPENDENT SET, given a graph $G(V, E)$, we are asked to determine a minimum size vertex-subset that is simultaneously independent and dominating. This problem, although polynomially solvable in strongly chordal graphs [14], has been proved to be inapproximable within $n^{1-\varepsilon}$, for any $\epsilon > 0$, not only in general graphs [19] but also in restricted graph classes as, for instance, the class of circle graphs [10]. Also, and probably due to this fact, exact solution of MIN INDEPENDENT DOMINATING SET in general or in restricted classes of graphs by moderately exponential algorithms has received growing attention in the past years [18,17,6]. This problem has also been tackled using exponential approximation techniques [6]. Finally, it is shown to be very hard from a parameterized complexity point of view since it is **W[2]**-hard [11].

Surprisingly, the natural symmetric problem, the MAX MIN VERTEX COVER problem, where the goal is to compute a minimal (for exclusion) vertex cover of maximum size, has received very little attention. To the best of our knowledge, the only result known about it, is an FPT algorithm [16] running in time $O^*(2^{\mathrm{opt}(G)})$, where $\mathrm{opt}(G)$ denotes the size of the optimum (also called standard parameter) and notation $O^*(\cdot)$ ignores polynomial factors.

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Given the direct applications of MIN INDEPENDENT DOMINATING SET in terms of ad-hoc wireless networks, it seems natural to study the symmetric version, where instead of minimizing the number of servers, one wishes to maximize the number of clients. This problem obviously has the same characteristics as its minimization counterpart in terms of **NP**-hardness and exact computation, but might have different behaviors in terms of approximability and parameterized complexity (as in the case of the pair MAX INDEPENDENT SET–MIN VERTEX COVER [4,11]).

2. Preliminaries

In what follows, given a graph $G(V, E)$ we denote by n its size (the cardinality of its vertex-set V). For a vertex $v_i \in V$, $\Gamma(v_i)$ denotes the set of its neighbors, d_i the cardinality of $\Gamma(v_i)$, i.e., the degree of a vertex v_i. Also, Δ denotes the maximum degree of G and d the average degree of G. For a set $S \subseteq V$, $\Gamma(S)$ denotes the set $\{v \in V \setminus S : v \in \Gamma(u) \text{ for some } u \in S\}$.

Given a matching M of G, we call the vertices of $V(M)$ (the set of endpoints of the edges in M) matched vertices and, using the terminology of [23], we call the vertices of $V \setminus V(M)$ exposed vertices (with respect to M). Throughout the paper we use m to denote the cardinality of M, and we set $P = V \setminus V(M)$ and $p = |P|$.

Given $V' \subseteq V$, we denote by $G[V']$ the subgraph of G induced by V'. For simplicity, edges having common endpoints are called adjacent, while edges without common endpoints are called disjoint.

For a problem Π defined on G, we denote by SOL(G) a feasible solution for Π in G (computed by some approximation algorithm), by sol(G) the value of SOL(G), by OPT(G) an optimal solution for Π in G and by opt(G) its value.

Throughout the paper exponential or parameterized complexity is expressed using notation $O^*(\cdot)$ that ignores polynomial factors.

We show in this paper that, while also being highly inapproximable, MAX MIN VERTEX COVER is better approximable than its mate, since it can be approximately solved in polynomial time within ratio $n^{-1/2}$. This result is matched by an inapproximability bound of $n^{\varepsilon-(1/2)}$ that can be extended to an $O(1/\Delta)$ inapproximability bound. We also match it to an asymptotic $(2/\Delta)$-approximation ratio. We then study parameterized complexity of MAX MIN VERTEX COVER. We first improve, the fixed-parameter tractability result by [16], with respect to the standard parameter, showing that there exists an FPT algorithm for MAX MIN VERTEX COVER that runs in time $O^*(1.5397^{\text{opt}(G)})$. We also study parameterization of MAX MIN VERTEX COVER by the cardinality of a maximum matching (that is smaller than the value of the optimum) and prove that the problem still remains FPT.

Finally, let us note that both MIN WEIGHTED DOMINATING SET and MAX WEIGHTED INDEPENDENT SET are polynomially solvable in graphs with bounded treewidth [2,3,5] (and, actually, fixed parameter tractable with respect to the treewidth of the input graph [24]). With similar dynamic programming techniques, it can be shown that also both WEIGHTED MAX MIN VERTEX COVER and WEIGHTED MIN INDEPENDENT DOMINATING SET are fixed parameter tractable with respect to the treewidth. Since the techniques used for obtaining this result are quite similar to those in [24], the related proof is omitted.

A preliminary version of this work was presented at WAOA 2013 [21]. Here, we extend those results and in particular we improve:

- the approximation ratio (with respect to Δ) from $3/2\Delta$ to $2/\Delta$ (Theorem 3);
- the complexity of the fixed parameter algorithm (with respect to the standard parameter) from $O^*(1.5874^{\text{opt}(G)})$ to $O^*(1.5397^{\text{opt}(G)})$ (Theorem 4);
- the complexity of the fixed parameter algorithm (with respect to the cardinality of a maximum matching M of the input graph) from $O^*(3^m)$ to $O^*(2.8284^m)$ (Theorem 5).

Furthermore, we have added Section 5 where we study moderately exponential and parameterized approximation issues. In particular, we study parameterized approximation of MAX MIN VERTEX COVER with respect to another parameter, and the cardinality of the set P of exposed vertices with respect to a maximum matching M. As we will see in Section 3, this parameter is quite meaningful for the approximation of MAX MIN VERTEX COVER, since this problem is better-approximable in graphs with large maximum matching (and consequently with small p) than in graphs with small maximum matching (and large p).

3. Approximability of the MAX MIN VERTEX COVER problem

We give in this section inapproximability upper bounds matched by lower bounds achieved in polynomial time for MAX MIN VERTEX COVER.

3.1. (In)Approximability within $\Theta(n^{-1/2})$

We first prove in this section that MAX MIN VERTEX COVER cannot be approximated in polynomial time within the ratio $\Omega(n^{-1/2})$. Then, we match this bound with a $O(n^{-1/2})$ approximation ratio.

Theorem 1. *For any positive constant ϵ,* MAX MIN VERTEX COVER *is inapproximable within the ratio $O(n^{\epsilon-(1/2)})$ unless* $\mathbf{P} = \mathbf{NP}$.

Proof. First, recall that MAX INDEPENDENT SET has been proved to be inapproximable within the ratio $n^{\varepsilon-1}$ for any given $\varepsilon > 0$ unless $\mathbf{P} = \mathbf{NP}$ [25].

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