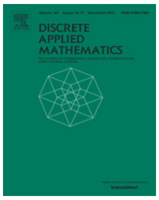
Discrete Applied Mathematics 182 (2015) 46–60

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam



Directed weighted improper coloring for cellular channel allocation

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ARTICLE INFO

Article history:
Received 28 December 2012
Received in revised form 11 November 2013
Accepted 16 November 2013
Available online 2 December 2013

Keywords:
Graph coloring
Weighted directed graphs
Channel assignment
Set partitioning formulations
Branch-and-price algorithm

ABSTRACT

Given a directed graph with weights on the vertices and on the arcs, a θ-improper k-coloring is an assignment of at most k different colors to the vertices of G such that the weight of every vertex v is greater, by a given factor $\frac{1}{\theta}$, than the sum of the weights on the arcs (u, v) entering v with the tail u of the same color as v. For a given real number θ, we consider the problem of determining the minimum integer k such that G has a θ-improper k-coloring. Also, for a given integer k, we consider the problem of determining the minimum real number θ such that G has a θ-improper k-coloring. We show that these two problems can be used to model channel allocation problems in wireless communication networks, when it is required that the power of the signal received at a base station is greater, by a given factor, than the sum of interfering powers received from mobiles which are assigned the same channel. We propose set partitioning formulations for both problems and describe branch-and-price algorithms to solve them. Computational experiments are reported for instances having a similar structure as real channel allocation problems.

1. Motivation

In wireless communication networks of the first and second generation, the concepts of cellular channel allocation and spatial frequency reuse were the key ideas that have driven the immense initial success of mobile telephony [13]. In this context, geographical regions were divided into cells, theoretically hexagonal, and each cell had a dedicated number of antennas with an associated set of frequency bands. For example, in the original AMPS, American Mobile Phone System, seven sets, which can be further subdivided into three subsets, were the original 'colors' with which one would allocate the channels to the regions (see Fig. 1(a) and (b)). In actual systems, many configuration of base stations are deployed according to the geo-demographic situation. In simple systems and in low density demographic areas, the base stations use omni-directional antennas and are thus viewed as being at the center of the cells 1(a). However in most systems, the base stations have three, 120° sectored, antennas, which permits to effectively discriminate the radio visibility horizon into three distinct parts. The base station is then viewed as being at the edges of three cells as in Fig. 1(b).

A similar scheme existed for GSM and the practical allocation problem consisted of selecting the set of channels associated with a cell or even subsets associated with an antenna. The criteria for channel selection consists in minimizing the mutual interference caused by transmission on the same channel in different cells or geographical regions. This may be reduced

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http://dx.doi.org/10.1016/j.dam.2013.11.018

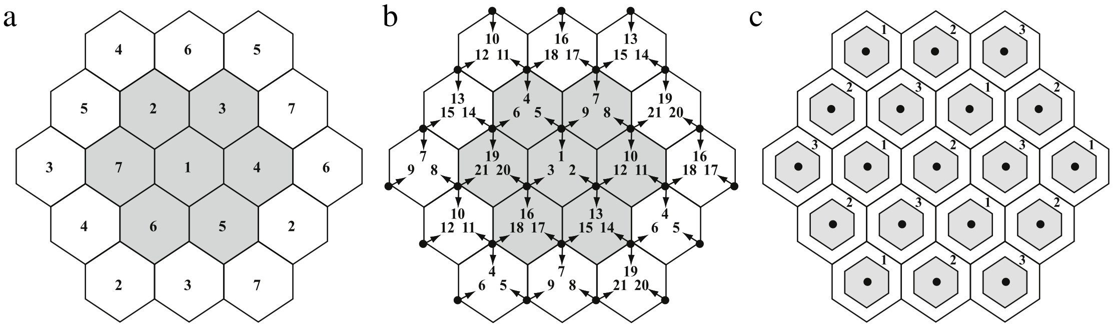

Fig. 1. Three different types of channel allocations.

to maximizing the geographical distance between channel reuse. With real world cells, which differ in size and shape, the problem typically required coloring algorithms to solve them [18,1].

In the third generation of mobile systems, the introduction of CDMA (Code Division Multiple Access) has enabled the reuse of the whole frequency band in each cell [13]. Instead of dividing the signal space in time or frequency between users, a code or pseudo-random sequence is used to differentiate the signal from each transmitter. Interference is tolerated up to a certain degree between transmitters within an area. In this context the coloring schemes and associated research projects were of much reduced importance.

In their latest incarnations, the fourth generation mobile standards mainly use Orthogonal Frequency Division Multiple Access schemes [12]. These schemes divide the signal space in time slots and orthogonal frequencies. At the middle of a cell, all slots of time and frequency are allocated to users. These cover the whole frequency band. At the edge of the cell, only part of the band is used and a three color scheme is used (see Fig. 1(c)): only a third of the whole frequency band is available at the edge of each cell to reduce interference. In these latest standards the base station uses sectored but also multiple antennas, which are jointly managed. The base station is thus usually viewed as being in the center of the cell. This is what we will assume in this paper. Furthermore, a form of spatial multiplexing may also be used within a cell by applying MIMO techniques (Multiple-Input Multiple-Output). This permits to allocate the same time–frequency slot to a number of users which are joined through quasi-independent paths. By storing the signal gain between each pair of antennas in a matrix, it is possible to perform some forms of diagonalization of this matrix to obtain independent transmission streams.

For these latest standards, the superposition of signals is tolerated to a certain degree [17]. That is, for a given signal that is denoted as the desired signal, one can tolerate the superposition of all other signals if their projection on the signal space–time is less powerful than the desired signal by a certain factor. Finally, when distances are relatively short, such as in dense urban areas, the random thermal noise, always present at a received antenna, becomes non-significant and the interference from other users or other cell sites dominate the communication performances. In this context, the capacity of the wireless communication scheme is said to be interference limited or dominated. The allocation of a time–frequency slot is akin to the allocation of a color. As a simplifying factor but without loss of generality, we will discount the angular separation of mobiles and compute the contribution of each mobile to the reception of the same colored signals at a base station as a function of distance.

More formally, let $M = \{m_1, \dots, m_n\}$ be a set of mobiles and $B = \{b_1, \dots, b_t\}$ a set of base stations. Let d_{ip} denote the Euclidean distance between the mobile m_i and the base station b_p. The positions are assumed here to be in two dimensions (in three dimensions one must account for a number of other factors such as floors). The received power P_{ip} at b_p of a signal from m_i is computed as follows:

$$P_{ip} = \frac{\alpha\, r_{ip}}{{d_{ip}}^{\gamma}}$$

where

- α is a constant summarizing the effects of antenna configuration and position,
- r_{ip} is a random variable synthesizing the desired channel models: it would typically be the product of an exponential random variable to denote short term fading and a log-normal random variable for shadowing [1],
- γ is the attenuation factor which typically varies from 2 in free-space to 4 for Non-Line Of Sight (NLOS) in urban areas.

Each mobile is assumed to be assigned to a given base station. Let $c(i)$ denote the channel, or color, assigned to mobile m_i. Also, for a channel k, let us denote by C_k the set of mobiles m_i assigned with channel $c(i) = k$. An admissible coloring scheme would require that the power of all received signals at their assigned base stations are greater, by a given factor $\frac{1}{\theta}$, than the sum of interfering powers received from mobiles which are assigned the same channel. More precisely, for a mobile m_i assigned to a base station b_p, we have the following constraint:

$$\sum_{m_j \in C_{c(i)}, j \neq i} P_{jp} \leq \theta\, P_{ip}. \tag{1}$$

where $\frac{1}{\theta}$ is the maximal admissible signal-to-interference ratio.