Discrete Applied Mathematics 177 (2014) 121–136

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Partial-route inequalities for the multi-vehicle routing problem with stochastic demands

Ola Jabali [a,*], Walter Rei [b], Michel Gendreau [c], Gilbert Laporte [a]

[a] CIRRELT and HEC Montréal 3000, chemin de la Côte-Sainte-Catherine, Montréal, Canada H3C 3J7

[b] CIRRELT and Department of Management and Technology, Université du Québec à Montréal, 315 rue Sainte-Catherine est, Montréal, Canada H2X 3X2

[c] CIRRELT and Department of Mathematics and Industrial Engineering, École Polytechnique de Montréal, C.P. 6128, succursale Centre-ville, Montréal, Canada H3C 3J7

ARTICLE INFO

Article history:
Received 29 November 2012
Received in revised form 20 February 2014
Accepted 22 May 2014
Available online 14 June 2014

Keywords:
Stochastic vehicle routing problem
Stochastic demand
Stochastic programming
Integer *L*-shaped algorithm
General partial routes
Lower bounding functionals

ABSTRACT

This paper describes an exact algorithm for a variant of the vehicle routing problem in which customer demands to be collected are stochastic. Demands are revealed upon the vehicle arrival at customer locations. As a result, a vehicle may reach a customer and does not have sufficient capacity to collect the realized demand. Such a situation is referred to as a failure. In this paper the following recourse action is then applied when failure occurs: the vehicle returns to the depot to unload and resumes its planned route at the point of failure. The capacitated vehicle routing problem with stochastic demands (VRPSD) consists of minimizing the sum of the planned routes cost and of the expected recourse cost. The VRPSD is formulated as a two-stage stochastic programming model and solved by means of an integer *L*-shaped algorithm. This paper introduces three lower bounding functionals based on the generation of general partial routes, as well as an exact separation procedure to identify violated cuts. Extensive computational results confirm the effectiveness of the proposed algorithm, as measured by a substantial reduction in the number of feasible solutions that have to be explicitly eliminated. This translates into a higher proportion of instances solved to optimality, reduced optimality gaps, and lower computing times.

1. Introduction

The aim of the vehicle routing problem (VRP) is to construct vehicle routes through a set of customers under side constraints. In the classical VRP, a travel cost matrix between customers is provided, several identical capacitated vehicles are available, and customers have demands to be collected. The routes start and end at the depot, each customer is visited exactly once by a single vehicle, and the demand collected on a route cannot exceed the vehicle capacity. The objective is to minimize the total travel cost.

The VRP has been extensively studied (see, e.g., Laporte [17]). A number of variants of the VRP have been proposed (see, e.g., Toth and Vigo [27] and Golden et al. [11]) to represent more realistic settings, and several efficient solution procedures have been developed for the VRP and its variants. Most of this research deals with deterministic settings, thus implicitly implying that all information concerning the instance parameters is known when the problem is solved. This assumption applies to situations where the estimated variability in the problem parameters is relatively low. However, in practice, data

* Corresponding author. Tel.: +1 5143406154; fax: +1 5143406834.
E-mail addresses: ola.jabali@hec.ca, Ola.Jabali@cirrelt.ca (O. Jabali), Walter.Rei@cirrelt.ca (W. Rei), Michel.Gendreau@cirrelt.ca (M. Gendreau), Gilbert.Laporte@cirrelt.ca (G. Laporte).

http://dx.doi.org/10.1016/j.dam.2014.05.040
0166-218X/

are often stochastic. In such contexts, solving a deterministic problem in which stochastic parameters are replaced by their expected values can yield poor solutions (Louveaux [21]). As a result, several stochastic versions of the VRP have been studied in recent years (Cordeau et al. [8]).

In this paper we solve the VRP with stochastic demands (VRPSD) in which demand is only revealed when the vehicle reaches the customer's location. Such problems occur in a number of applications, for example in the delivery and collection of money to and from banks (Bertsimas [4] and Lambert et al. [16]), in home oil delivery (Chepuri and Homem de Mello [23]), beer distribution and garbage collection (Yang et al. [29]).

Because demand is not known beforehand, a vehicle may reach a customer location with insufficient residual capacity to collect the observed demand. This causes a *route failure*, in which case a *recourse* action can be implemented. One of the most common solution frameworks for this class of problems is *a priori optimization*, a concept initially put forward by Bertsimas [5], Jaillet [14] and Bertsimas et al. [6]. It consists of modeling the problem in two stages. In the first stage, a planned, or *a priori* solution is designed, before customer demands are known. It consists of the set of vehicle routes. In the second stage, these routes are performed as demands are gradually revealed. Whenever a failure occurs, a predetermined recourse policy is implemented, which entails an extra recourse cost. The objective of the problem is to minimize the cost of the first-stage solution plus the expected cost of recourse.

Several recourse policies have been proposed for the VRPSD. A *classical* policy is to return to the depot upon failure, offload, and resume collections by following the planned route starting at the point of failure (Christiansen and Lysgaard [7], Gendreau et al. [9,10], Goodson et al. [12], Hjorring and Holt [13], Laporte et al. [19], Lei et al. [20] and Rei et al. [24]). More involved recourse policies have also been considered, such as restocking rules (Yang et al. [29]), route reoptimization (Secomandi and Margot [25]), pairing strategies (Ak and Erera [1]), and the use of safety stocks (Juan et al. [15]). Note that the algorithms described in Ak and Erera [1], Hjorring and Holt [13], Rei et al. [24] and Secomandi and Margot [25] have been implemented for the single-vehicle case only. An important managerial advantage of the classical policy is that it yields stable routes which require minimal alterations in the event of failure. Solving the VRPSD under the classical recourse policy also provides a benchmark against which alternative policies can be assessed. Whereas most authors in the field of stochastic vehicle routing cast the problem in the context of stochastic programming, one study by Sungur et al. [26] defines it in the context of robust optimization which yields routes that minimize transportation costs while satisfying all the demands in a given bounded uncertainty set.

Compared with the classical VRP, the VRPSD is considerably more difficult to solve. For example, state-of-the-art algorithms for the VRP can handle instances involving up to 200 customers and 17 vehicles (Baldacci et al. [2]). In contrast, the best available algorithms for the VRPSD (with multiple vehicles) can only handle instances with 50 customers and three vehicles under a normal demand distribution (Laporte et al. [19]).

As in Laporte et al. [19], we formulate the VRPSD as a two-stage stochastic programming model under the classical recourse policy. We solve it optimally by means of the integer *L*-shaped method proposed by Laporte and Louveaux [18], an extension of the *L*-shaped method of Van Slyke and Wets [28] for continuous stochastic programs, itself an application of Benders decomposition [3] to stochastic programming. The integer *L*-shaped method follows a branch-and-cut framework in which lower optimality cuts are generated to eliminate feasible solutions, and lower bounding functionals (LBFs) are commonly used to improve the efficiency of the algorithm. Their role is to tighten the linear relaxation of the current subproblem, thus stemming the growth of the search tree. When applied to the VRPSD, LBFs are used to strengthen the lower bounds on the recourse cost associated to partial routes encountered throughout the solution process. As was numerically illustrated by Hjorring and Holt [13] and Laporte et al. [19], the use of LBFs is instrumental in optimally solving the VRPSD.

This paper makes three main scientific contributions. It first generalizes the concept of partial routes defined by Hjorring and Holt [13] for the single vehicle case, and by Laporte et al. [19] for the multi-vehicle case. It then proposes strengthened LBFs based on these generalized partial routes. Finally, it describes an exact separation algorithm for these LBFs. Extensive computational experiments on benchmark instances demonstrate that the combination of these improvements yields shorter computing times, thus enabling solving to optimality larger instances than what was previously possible. Moreover, it yields smaller optimality gaps on the unsolved instances.

The remainder of this paper is organized as follows. In Section 2 we present our modeling and solution framework for the VRPSD. Section 3 introduces the generalized definition of partial routes. These are used to generate stronger LBFs in Section 4. The exact separation procedure for the LBFs is presented in Section 5. This is followed by computational results in Section 6 and by conclusions in Section 7.

2. Model and algorithmic framework

We recall in Section 2.1 the two-stage stochastic programming formulation of VRPSD initially proposed by Laporte et al. [19], which constitutes the backbone of our solution framework. We then describe in Section 2.2 the integer *L*-shaped algorithm for which we introduce improvements in Sections 3–5.

2.1. The VRPSD model

The VRPSD is defined on a complete undirected graph $G = (V, E)$, where $V = \{v_1, \ldots, v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V,\ i < j\}$ is the edge set. Vertex v_1 is the depot at which m identical vehicles of capacity D are based,