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On the recognition of unit disk graphs and the Distance Geometry Problem with Ranges ☆

Guilherme Dias da Fonseca [a], Vinícius Gusmão Pereira de Sá [a,*], Raphael Carlos Santos Machado [b], Celina Miraglia Herrera de Figueiredo [a]

[a] *Universidade Federal do Rio de Janeiro, Brazil*

[b] *Instituto Nacional de Metrologia, Qualidade e Tecnologia, Brazil*

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ABSTRACT

We introduce a method to decide whether a graph G admits a realization on the plane in which two vertices lie within unitary distance from one another exactly if they are neighbors in G. Such graphs are called unit disk graphs, and their recognition is a known **NP**-hard problem. By iteratively discretizing the plane, we build a sequence of geometrically defined trigraphs – graphs with mandatory, forbidden and optional adjacencies – until either we find a realization of G or the interruption of such a sequence certifies that no realization exists. Additionally, we consider the proposed method in the scope of the more general Distance Geometry Problem with Ranges, where arbitrary intervals of pairwise distances are allowed.

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1. Introduction

The fundamental *Distance Geometry Problem* (DGP) consists in determining a set of points whose pairwise distances are known *a priori*. More formally, the input comprises a graph where the weight of each edge indicates the exact intended distance between its incident vertices in a realization of the graph in some metric space.

The generality of the DGP embraces a variety of applications, notably in wireless sensor networks, pattern recognition, computational biology and astronomy, to cite but a few. A survey on the theory of Euclidean Distance Geometry and its most important applications, with special emphasis to molecular conformation problems, can be found in [6]. Recently, researchers of multidisciplinary fields were brought together in a workshop dedicated to the theme [1], where a preliminary version of this paper was presented.

We consider a generalization of the DGP where the exact distances between vertices are replaced by intervals. We call it the *Distance Geometry Problem with Ranges* (DGPR). The applicability of such a generalized model is clear. Distance ranges may be due to imprecisions – for example, in measuring intermolecular distances – or to natural relaxations of exact distances into intervals of allowed distances – for instance, in the deployment of a wireless telecommunication infrastructure, where some antennas should be neither too close (to avoid interference) nor too far from one another (to allow for a control communication channel between them).

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* Corresponding author. Tel.: +55 21 81069166; fax: +55 21 25989515.
E-mail addresses: fonseca@cos.ufrj.br (G.D. da Fonseca), vigusmao@dcc.ufrj.br, vigusmao@gmail.com (V.G. Pereira de Sá), rcmachado@inmetro.gov.br (R.C.S. Machado), celina@cos.ufrj.br (C.M.H. de Figueiredo).

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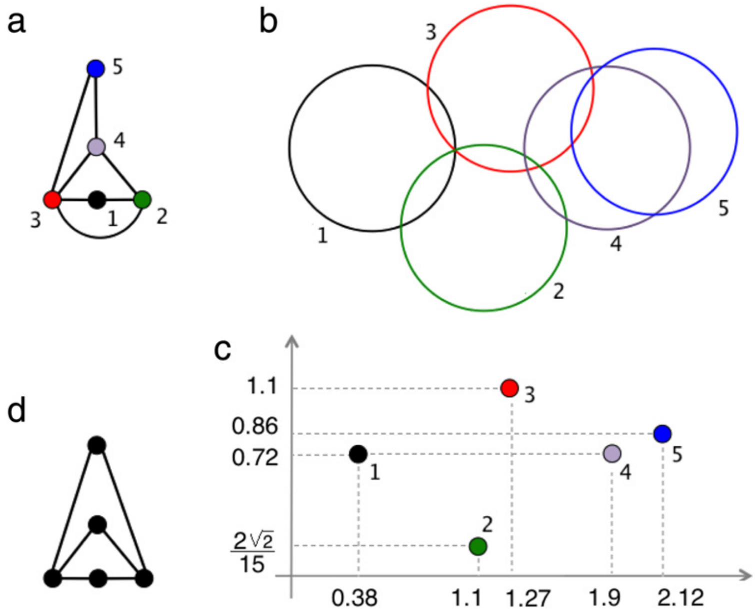

Fig. 1. (a) A unit disk graph G; (b) a model of congruent disks for G; (c) a formal UDG realization of G; (d) the graph $K_{2,3}$, which is not a UDG.

First and foremost, we focus on the case where each range is either $[0, 1]$ or $(1, \infty)$. Given such restriction, the problem corresponds exactly to a classic **NP**-hard problem in Graph Theory, namely the recognition of unit disk graphs. The technique we introduce, based on an iterative discretization of the continuous solution space along with a tripartition of the edges of an auxiliary graph, is nevertheless general enough to suit the (unrestricted) DGPR with only minor modifications.

This paper is organized as follows. In Section 2, we give a brief account on unit disk graphs, discussing the importance and the difficulty of recognizing them. In Section 3, we introduce the concept of trigraph realizations, which is central in our discretization technique. In Section 4, we propose a method for unit disk graph recognition which has proved to work well for graphs with a small number of vertices. In Section 5, we present some experimental results, including the unprecedented computational recognition of several graphs. In Section 6, we consider the prospects of leveraging our technique to the more general scope of the DGPR, posing a series of open theoretical questions. In Section 7, we make our concluding remarks.

Throughout the text, we denote the vertex set and the edge set of a graph G respectively as $V(G)$ and $E(G)$, as usual. The set of neighbors of v is denoted $N(v)$, with $d(v) = |N(v)|$. An edge incident to vertices v and w is written vw, or, if v or w are numbers, $\{v, w\}$. All norms $\|\cdot\|$ are Euclidean.

2. Unit disk graphs

A *unit disk graph* (UDG) is a graph whose vertices can be mapped to points on the plane and whose edges are defined by pairs of points within unitary Euclidean distance from one another. Unit disk graphs can also be regarded as intersection graphs of coplanar congruent disks, and they have been widely studied in recent times owing to their relevance in wireless sensor networks [7].

Definition 1. Given a graph G, a *UDG realization* of G is a function $\phi: V(G) \to \mathbb{R}^2$ such that, for some real number $d > 0$ and all $u, v \in V(G)$,

- $\|\phi(u) - \phi(v)\| \leq d$, if $uv \in E(G)$; and
- $\|\phi(u) - \phi(v)\| > d$, if $uv \notin E(G)$.

A graph is a UDG if and only if it admits a UDG realization. For convenience, the value of d is often taken to be 1, hence the name "unit" disk graph. In this paper, we consider it to be so, unless marked otherwise. The graph in Fig. 1(a) is a UDG, whereas the graph in Fig. 1(d) is not.

The problem of recognizing whether a given graph is a UDG is **NP**-hard [2], and the fastest known recognition algorithm is doubly exponential [9]. It is not known for the time being whether the problem belongs to **NP**, since the size of the natural certificate comprising the coordinates of the vertices (i.e., a UDG realization) is not polynomially bounded under the classic model of computation over finite strings [8], and no other polynomial certificates are known.

Whereas many applications would benefit from the ability to obtain feasible UDG realizations, being able to prove that some graphs are *not* UDG would also have immediate consequences. For example, in [11], Wu et al. conjecture that the graph in Fig. 2(a) is not a UDG. The correctness of such conjecture would imply a decrease from 3.8 to 3.6 in the maximum

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