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3D well-composed polyhedral complexes

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ABSTRACT

A binary three-dimensional (3D) image I is well-composed if the boundary surface of its continuous analog is a 2D manifold. In this paper, we present a method to locally "repair" the cubical complex $Q(I)$ (embedded in $\mathbb{R}^3$) associated to I to obtain a polyhedral complex $P(I)$ homotopy equivalent to $Q(I)$ such that the boundary surface of $P(I)$ is a 2D manifold (and, hence, $P(I)$ is a well-composed polyhedral complex). For this aim, we develop a new codification system for a complex K, called ExtendedCubeMap (ECM) representation of K, that codifies: (1) the information of the cells of K (including geometric information), under the form of a 3D grayscale image g_P; and (2) the boundary face relations between the cells of K, under the form of a set B_P of structuring elements that can be stored as indexes in a look-up table. We describe a procedure to locally modify the ECM representation E_Q of the cubical complex $Q(I)$ to obtain an ECM representation of a well-composed polyhedral complex $P(I)$ that is homotopy equivalent to $Q(I)$. The construction of the polyhedral complex $P(I)$ is accomplished for proving the results though it is not necessary to be done in practice, since the image g_P (obtained by the repairing process on E_Q) together with the set B_P codify all the geometric and topological information of $P(I)$.

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1. Introduction

3D well-composed images [12] enjoy important topological and geometrical properties in such a way that several algorithms used in computer vision, computer graphics and image processing are simpler. For example, thinning algorithms can be simplified and naturally made parallel if the input image is well-composed [14,16]; some algorithms for computing surface curvature or extracting adaptive triangulated surfaces assume that the input image is well-composed [10]. However, our main motivation is that of (co)homology computations on the cell complex representing the 3D image [3,5]. We could take advantage of a well-composed-like representation since such computations could be performed only on the boundary subcomplex.

3D images are often not well-composed. There are several methods (repairing algorithms) for turning them into well-composed ones [13,18]. These methods do not guarantee the topological equivalence between the original and its corresponding well-composed image. In fact, the purpose may even be to simplify as much as possible the topology (in the sense of removing little topological artifacts). However, we are concerned with the fact of preserving the topology of the input image having in mind cases in which subtle details may be important.

In [20], a solution to the problem of topology preservation during digitization of 3D objects is provided using several reconstruction methods that result in a 3D object with a 2D manifold surface. More specifically, one of the proposed

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methods is a voxel-based method called Majority Interpolation, by which resolution is doubled in any direction and new sampling voxels are added to the foreground under some constraints. Another method is based on the most common reconstruction method for 3D digital images, the marching cubes (MC) algorithm [15], which analyzes local configurations of eight neighboring sampling points in order to reconstruct a polygonal surface. There even exists a MC variant, called asymmetric marching cubes, which generates the reconstruction of manifold surfaces (see [19, page 101]). A different approach is made in [11], where the authors create a polyhedral complex as the continuous analog of a set of voxels with given digital adjacencies. They also show that such a continuous analog corresponds to the usual definition of iso-surface in the 3D case.

In our approach, we first consider a cubical complex $Q(I)$ associated to a voxel-based representation of the given image I. Then, we develop a new scheme of representation, called *ExtendedCubeMap (ECM) representation*, based on a 3D grayscale image storing the cells and the boundary face relations between cells of $Q(I)$. This data structure extends the one called *CubMap representation*, from [21,2], to store and manipulate cubical complexes. Working on an ECM representation of $Q(I)$, we design a procedure to obtain a polyhedral complex $P(I)$ homotopy equivalent to $Q(I)$. The novelty of our method is that the cells of $P(I)$ are totally encoded in a 3D grayscale image g_P, and their boundary face relations in a so-called *set of structuring elements* B_P, which describe all the local configurations in g_P that provide the boundary faces of each cell of $P(I)$. Therefore, we do not need to compute $P(I)$ to manipulate cells and boundary relations between cells. It is worth to mention that the set B_P remains the same for any polyhedral complex $P(I)$ computed using our method. As far as we know, this is the first time that polyhedral complexes more general than cubical complexes are computed and stored using grayscale images.

In our prequel paper [4], the complex $P(I)$, homotopy equivalent to $Q(I)$, was a cell complex constructed with more general building blocks than polyhedra and the process depended on the local configuration of voxels. In this paper, $P(I)$ is always a polyhedral complex, constructed with a general procedure that is valid for all the local configurations, with the advantage that $P(I)$ is totally encoded in a matrix form (a 3D grayscale image) in a way that we do not need to build $P(I)$ to obtain geometric and topological information of $P(I)$ such as position of the vertices or face relations between cells.

Section 2 is devoted to clarify, first, the correspondence between 3D binary digital images and cubical complexes. The notion of well-composedness is also introduced as well as its extension to complete polyhedral complexes. Section 3 describes a new codification system called ECM representation of cubical complexes which is also valid for other more general polyhedral complexes as we will see later. Section 4 describes the repairing algorithm to get a well-composed polyhedral complex, $P(I)$, starting from the cubical complex $Q(I)$ associated to a non-well-composed image I. The repairing process is, in fact, accomplished on an ECM representation E_Q of $Q(I)$ to get an ECM representation E_P of $P(I)$. Finally, we draw some conclusions and statements for future work in Section 5 and provide tables summarizing the symbols used in the different.

2. Digital images, polyhedral complexes and well-composedness

Consider $\mathbb{Z}^3$ as the set of points with integer coordinates in 3D space $\mathbb{R}^3$. A *3D binary digital image* (or 3D image for short) is a set $I = (\mathbb{Z}^3, 26, 6, B)$ (or $I = (\mathbb{Z}^3, B)$, for short), where $B \subset \mathbb{Z}^3$ is the foreground, $B^c = \mathbb{Z}^3 \setminus B$ the background, and (26, 6) is the adjacency relation for the foreground and background, respectively. A point $p \in \mathbb{Z}^3$ can be interpreted as a unit closed cube (called *voxel*) in $\mathbb{R}^3$ centered at p with faces parallel to the coordinate planes. The set of voxels centered at the points of B (the foreground) is called the *continuous analog* of I and it is denoted by $C(I)$. The *boundary surface* of $C(I)$ is the set of points in $\mathbb{R}^3$ that are shared by the voxels centered at points of B and those centered at points of B^c (see [1,7,17]). The set of voxels of $C(I)$ (considered as a set of cubes, squares, edges and vertices), constitutes a combinatorial structure called *cubical complex*, denoted by $Q(I)$, whose geometric realization is exactly $C(I)$. Let r_σ denote the barycenter of $\sigma \in Q(I)$. Observe that $Q(I)$ is constructed by adding unit cubes σ centered at points $r_\sigma = (i, j, k) \in \mathbb{Z}^3$. The possible coordinates of r_μ for the ℓ-faces μ of σ are:

- $(i \pm \frac{1}{2}, j \pm \frac{1}{2}, k \pm \frac{1}{2})$ if $\ell = 0$;
- $(i, j \pm \frac{1}{2}, k \pm \frac{1}{2})$, $(i \pm \frac{1}{2}, j, k \pm \frac{1}{2})$ or $(i \pm \frac{1}{2}, j \pm \frac{1}{2}, k)$, if $\ell = 1$;
- $(i \pm \frac{1}{2}, j, k)$, $(i, j \pm \frac{1}{2}, k)$ or $(i, j, k \pm \frac{1}{2})$, if $\ell = 2$.

Therefore, for any $\mu \in Q(I)$, $r_\mu = (i, j, k)$ has integer multiple-of$-\frac{1}{2}$ coordinates (i.e., $(i, j, k) \in \frac{1}{2}\mathbb{Z}^3$).

Recall that a 3D image $I = (\mathbb{Z}^3, B)$ is *well-composed* [12] if the *boundary surface* of $C(I)$ is a 2D manifold, i.e. each point has a neighborhood homeomorphic to $\mathbb{R}^2$ (it "looks" locally like a planar open set). Since the topology of $Q(I)$ reflects the topology of I whenever (26, 6)-adjacency is considered on I, we will say that $Q(I)$ *is well-composed* if the corresponding image I is well-composed. Let an 8-*cube configuration* be the set of eight cubes sharing a common vertex. One finds eleven different 8-cube configurations (modulo reflections and 90° rotations) that will be called *critical configurations*, where well-composedness condition is not satisfied (see Fig. 1). For any critical configuration, the central vertex v is a *critical vertex* (it does not have a neighborhood in the boundary surface of $Q(I)$ homeomorphic to $\mathbb{R}^2$). These eleven critical configurations come exactly from the just two presented in [12, Fig. 3].

A cubical complex is a specific type of *polyhedral complex* (see [9]). A polyhedral complex K is a combinatorial structure by which a space is decomposed into vertices, edges, polygons and polyhedra (cells, in general) that are glued together by their boundaries such that the intersection of any two cells of the complex is also a cell of the complex. Notice that the structure of

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