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Remainder approach for the computation of digital straight line subsegment characteristics

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ABSTRACT

Given a digital straight line $\mathcal{D}$ of known characteristics (a, b, c), and given two arbitrary discrete points $A(x_a, y_a)$ and $B(x_b, y_b)$ of it, we are interested in computing the characteristics of the digital straight segment (DSS) of $\mathcal{D}$ delimited by the endpoints A and B. Our method is based entirely on the remainder subsequence $S = \{ax - c \bmod b;\ x_a \leq x \leq x_b\}$. We show that minimum and maximum remainders correspond to the three leaning points of the subsegment needed to determine its characteristics. One of the key aspects of the method is that we show that computing such a minimum and maximum of a remainder sequence can be done in logarithmic time with an algorithm akin to the Euclidean algorithm. Experiments show that our algorithm is faster than the previous ones proposed in Lachaud and Said (2013) and in Sivignon (2013).

1. Introduction

One of the simplest primitives in digital geometry, the Digital Straight Line (DSL) and the Digital Straight line Segment (DSS), have very interesting and rich structures that have been studied for a long time now [14]. See [10] for a historical overview. There are immediate links to Sturmian and Christoffel words, the Stern–Brocot tree, the Farey fans, etc. [10]. The study regained some interest when J-P. Reveillés, proposed, among previous authors [3,5], an analytical description of a DSL $0 \leq ax - by - c < \omega$ in [15] (where (a, b, c) are called the characteristics or parameters of the DSL). The immediate possibilities of extensions to higher dimensions and to different scales sparked interest among arithmeticians [2] and researchers in image processing [17,20].

In this paper we are interested in a particular class of DSS recognition problems. The problem is related to multiscale geometry [17,20]. It is indeed important, when inspecting geometrical features at multiple scales, to be able to recompute the new, scaled, characteristics and that, as rapidly as possible. The intricate structure of DSLs is at the heart of many DSS recognition algorithms. While there are too many papers to cite them all, let us just recall some emblematic papers that did explore various approaches such as those based on convex hull computation [1,4], on arithmetic properties [12] or on pre-images of a digital straight line segment [9,19]. In [7], I. Debled-Rennesson proposed an algorithm that allows to compute the

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characteristics of a DSS in linear time complexity with a very simple straight forward algorithm that made the link between the characteristics of a DSS and leaning points which can be described arithmetically as points with limit remainder values or geometrically as pivot points. Let us consider a DSL with known characteristics (a, b, c) and two points A and B of the DSL. What are the characteristics of the DSS defined by those two points? We may consider a DSS as a segment of DSL at various scales and being able to characterize those DSSs allows multiscale shape analysis for example. The problem differs from the classical DSL recognition problem because we have a very important information we usually do not have: the whole set of points of the DSS belongs to a known DSL. This leads to algorithms in logarithmic time which is, of course, not possible if we do not know in advance which points belong to the DSS [13,18]. In classical Euclidean geometry, the characteristics of a segment are the same as those of the straight line. This is not true in digital geometry. There can be an infinite number of different triplets of DSS characteristics for a same set of points (using a simple Greatest Common Divisor is not enough). Although there is an infinite set of characteristics, there is a unique minimal characteristic triplet. For naive DSS ($\omega = b,\ 0 \leq a \leq b$) it corresponds to the characteristics with the minimal b.

This problem has been the focus of some attention lately in conjunction with new multiscale shape analysis methods [13,17], and various approaches have been tried: in [6], De Vieilleville and Lachaud exhibited relations describing the possible changes in the characteristics of a DSS by examining its combinatoric description. They established new analytic relations and made explicit the relation with the Stern–Brocot tree. In [13] Said and Lachaud used these results and presented two algorithms. Those determine the minimal characteristics of a DSS by moving in a bottom-up and top-down way along the Stern–Brocot tree. They demonstrated that the worst-time complexity is proportional to the difference of depth, in the Stern–Brocot tree, of the slope of the input line and the slope of the output segment and is thus logarithmic in the coefficients of the input slope. Sivignon in [18] proposed a method that computes the characteristics of a DSS using a walk in the so-called Farey Fans; this algorithm is logarithmic in terms of the length of the subsegment. The main problem with both these methods, aside from the fact that they are not trivial to program, is that they do not offer an obvious extension to higher dimensions.

In this paper we propose a new algorithm for the computation of the minimal characteristics of a DSS defined as a subsegment of a DSL with known characteristics. Our approach is entirely based on the remainders of the DSL points. For a DSL defined by $0 \leq ax - by - c < b$ (with $0 \leq a \leq b$), the remainder is simply the value $\mathcal{R}_{a,b,c}(x) = ax - by - c = \left\{\frac{ax-c}{b}\right\}$ where $\left\{\frac{n}{m}\right\}$ stands for $n \mod m$ (y is a function of x; there is one and only one DSL point per abscissa). We show that there is, under some conditions, an order relationship between the remainders of a point relatively to the DSL and to the DSS minimal characteristics. We show especially that the points with minimum and maximum DSL remainders are leaning points of the DSS. The third leaning point is obtained in a similar way on a sub-interval. The second important result of the paper is that the minimum and maximum of a remainder sequence can be computed in logarithmic time with a very simple algorithm that is akin to the Euclidean algorithm. Determining the three leaning points of a DSS that allow us to determine its characteristics is resumed by searching three times for a minima or maxima in remainder sequences. The resulting algorithm is very simple and efficient, being significantly faster as previous methods [13,18]. An interesting aspect of this approach is that it offers a new way, with remainders, to explore higher dimensions. It is not however, as can be seen in the conclusion, straightforward.

This paper is organized as follows: Section 2, deals with remainders and the properties of minimal DSS. Section 3 is devoted to the computation of the minimum and maximum of a remainder sequence. Section 4 presents briefly the algorithm for DSS characterization and shows some results and some comparisons of our approach with previous ones. Finally Section 5 proposes a conclusion and some perspectives.

2. Remainders of minimal DSS

In this section we present our notations, definitions and properties of Digital Straight Segments. We are especially going to explore the properties of the remainders of minimal DSS and relations to the remainders of corresponding DSL.

2.1. Notations, definitions

A *Digital Straight Line* (DSL for short) $\mathcal{D}\ (a, b, c)$ of *integer characteristics* (a, b, c) is the set of digital points $(x, y) \in \mathbb{Z}^2$ such that $0 \leq ax - by - c < \max(|a|, |b|)$ with $\gcd(a, b) = 1$. These DSL are 8-connected and called naive DSL [7]. The slope of the DSL is the fraction $\frac{a}{b}$. The value c is sometimes called the translation constant. In this paper, without loss of generality, we assume that $0 \leq a \leq b$. This corresponds to a DSL in the first octant with slopes $0 \leq \frac{a}{b} \leq 1$. In this case, we have one and only one point, denoted $P_D(x)$, in D with abscissa x. The ordinate is then $y = \left\lfloor \frac{ax-c}{b} \right\rfloor$.

A DSL can also be defined as the integer points of a strip delimited by the *lower leaning line* $\mathcal{D}_L : ax - by - c = b - 1$ and the *upper leaning line* $\mathcal{D}_U : ax - by - c = 0$ [7]. *Upper* (resp. *Lower*) *leaning points* are the digital points of the DSL lying on the upper (resp. lower) leaning lines. A weakly exterior point is a point of D that verifies $ax - by - c = -1$ (in this case we speak also of a weakly upper exterior point) or $ax - by - c = b$ (in this case we speak also of a weakly lower exterior point).

A *Digital Straight Segment* (DSS for short) $\mathcal{S}\ (D, u, v)$ associated to the DSL $D = \mathcal{D}\ (a, b, c)$ and *endpoints* $P_D(u)$ and $P_D(v)$ is the subset of D with points of abscissa in $[u, v]$. A DSS is a finite 8-connected subset of a DSL.