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Fast recognition of a Digital Straight Line subsegment: Two algorithms of logarithmic time complexity

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ABSTRACT

Given a Digital Straight Line (DSL) of known characteristics (a, b, μ), we address the problem of computing the characteristics of any of its subsegments. We propose two new algorithms that use the fact that a digital straight segment (DSS) can be defined by its set of separating lines. The representation of this set in the $\mathbb{Z}^2$ space leads to a first algorithm of logarithmic time complexity. This algorithm precises and extends existing results for DSS recognition algorithms. The other algorithm uses the dual representation of the set of separating lines. It consists of a smart walk in the so called Farey fan, which can be seen as the representation of all the possible sets of separating lines for DSSs. Indeed, we take profit of the fact that the Farey fan of order n represents in a certain way all the digital segments of length n. The computation of the characteristics of a DSL subsegment is then equivalent to the localization of a point in the Farey fan. Using fine arithmetical properties of the fan, we design a fast algorithm of theoretical complexity $\mathcal{O}(\log(n))$ where n is the length of the subsegment. Experiments show that our algorithms are also efficient in practice, with a comparison to the ones previously proposed by Lachaud and Said (2013): in particular, the second one is faster in the case of "small" segments.

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1. Introduction

Digital Straight Lines (DSL) and Digital Straight Segments (DSS) have been known for many years to be interesting tools for digital curve and shape analysis. The applications range from simple coding to complex multiresolution analysis and geometric estimators (see for instance [14] for a recent example). All these applications require to solve the so called DSS recognition problem. Many algorithms, using arithmetics [6], combinatorics [30] or dual-space [9] have been proposed to solve this problem, reaching a computational complexity of $\mathcal{O}(n)$ for a DSS of length n (see also [23] for an overview).

When no further information is known, all these algorithms are actually optimal. They at the same time decide if the set of grid points is a DSS and compute its characteristics (minimal in some sense). However, we sometimes know beforehand that the set of grid points is a DSS: the algorithm does not need to decide anymore and we can hope for a sublinear-in-time recognition algorithm. For instance, this extra information may come from the knowledge of the characteristics of a DSL containing the set of grid points. This occurs for example in [27] where the multiresolution geometry of a digital object is considered. Another example concerns the digitization of a straight segment on a grid of given size: we know that the set of grid points is a DSS, but its characteristics may be much smaller than the ones of the input straight segment (and not greater than the grid size).

In [27], the authors introduce the following problem: given a DSL of known characteristics and a subsegment of this DSL, compute the minimal characteristics of the DSS. The authors present two algorithms (SmartDSS and ReversedSmartDSS)

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to solve this problem in [27,26,19]: both use the decomposition into continuous fractions of the DSL slope and reach a logarithmic complexity.

However, a deeper search in the state-of-the-art shows that this problem is not so new. Indeed, in [20], the author presents a quick sketch of a method that solves it using the Farey fan. The announced complexity of the method is $\mathcal{O}(\log^2 n)$ for a segment of length n. Much later, the authors of [2], try to compute the "reduction" of a straight line, which is a simplification of DSL characteristics over a bounded domain. As we will see, this reduction does not compute the minimal characteristics, but the idea is similar.

Our contribution in this paper is to demonstrate that it is possible to solve the DSL subsegment problem in logarithmic time complexity revisiting and deepening the study of the two state-of-the-art algorithms [20,2].

The first algorithm is based on the local convex hull algorithm developed in [2] together with the framework for DSS recognition described in [24], but we provide the theoretical results which enable to efficiently compute the minimal characteristics of a subsegment from this hull. The second algorithm, detailed in section brings the method introduced in [20] up to date: we investigate it further to provide a thoroughly defined algorithm. Moreover, we show how its complexity can be lowered to $\mathcal{O}(\log(n))$ with an astute use of arithmetical properties of the Farey fan. The latter algorithm was first presented in [28] and a more detailed description is provided here.

Section 2 is dedicated to the presentation of the notions used in this work. Section 3 describes the algorithm based on the local convex hull computation. The algorithm using the dual representation, and more specifically the Farey fan is detailed in Section 4. At the end of this section, two extensions for slightly different frameworks are presented: in particular, we show that the second algorithm can be directly and reliably applied when the input data is not a DSL but a straight line with non integer parameters. The last section is classically devoted to experimental validations.

2. Preliminary definitions

2.1. Digital line, segment and minimal characteristics

A *Digital Straight Line* (DSL for short) of integer characteristics $(a, b, \mu) \in \mathbb{Z}^3$ is the infinite set of digital points $(x, y) \in \mathbb{Z}^2$ such that $0 \leq ax - by + \mu < \max(|a|, |b|)$, with a and b relatively prime [6]. These DSL are 8-connected and often called *naive*. The slope of the DSL is the fraction $\frac{a}{b}$ and $\frac{\mu}{b}$ is the shift at the origin. In the following, without loss of generality, we assume that $0 \leq a \leq b$, such that, on a DSL, there is exactly one pixel for each value of x. In this context, it is easy to see that the set of pixels of a given DSL is defined by a unique triplet (a, b, μ). The remainder of a DSL of characteristics (a, b, μ) for a given digital point (x, y) is the value $ax-by+\mu$. The *upper (resp. lower) leaning line* of a DSL is the straight line $ax-by+\mu = 0$ (resp. $ax - by + \mu = b - 1$). Upper (resp. lower) leaning points are the digital points of the DSL lying on the upper (resp. lower) leaning lines.

A *Digital Straight Segment* (DSS) is a finite 8-connected part of a DSL. It can be defined by the characteristics of a DSL containing it and two endpoints P and Q. However, a DSS belongs to an infinite number of DSLs. In this context, the *minimal characteristics* of a DSS are the characteristics of the DSL containing it with minimal b [29,19]. Since a DSL is defined by a unique triplet (a, b, μ), the values of a and μ are uniquely defined for a given b, and the minimal characteristics of a DSS are also uniquely defined.

Definition 1 (*Minimal Characteristics*)**.** Let $\mathcal{L} = \{(a, b, \mu) \in \mathbb{Z}^3, 0 \leq a \leq b, gcd(a, b) = 1\}$. For a given DSS S, we can define $\mathcal{L}_S = \{L \in \mathcal{L},$ the pixels of S all belong to the DSL of characteristics $L\}$. Let $f : \mathcal{L} \to \mathbb{Z}, f(a, b, \mu) = b$. Then the *minimal characteristics* of S is the triplet $(a_S, b_S, \mu_S) = \arg\min_{L\in\mathcal{L}_S} f(L)$.

Note that the notions of leaning points and lines are similarly defined for DSSs. DSS recognition algorithms aim at computing the minimal characteristics of a DSS, taking profit of the following fact: (a, b, μ) are the minimal characteristics of a DSS if and only if the DSS contains at least three leaning points [6]. In this case, the minimal characteristics are the characteristics of the DSS upper leaning line.

2.2. Minimal characteristics, separating lines and dual space

If we consider the digitization process related to this DSL definition, the points of the DSL **L** of parameters (a, b, μ) are simply the grid points (x, y) lying below or on the straight line $l : ax-by+\mu = 0$ (Object Boundary Quantization), and such that the points $(x, y + 1)$ lie above l. We say that **L** is the digitization of the straight line l. Note that **L** is also the digitization of all the straight lines of equation $ax - by + \rho = 0$ with $\mu \leq \rho < \mu + 1$, where $\rho \in \mathbb{R}$. These lines separate the points X of the DSL from the points $X + (0, 1)$, denoted by $\overline{X}$ (as in [24]), and they are called *separating lines*. Fig. 1(a) illustrates the separating lines of a DSL.

A similar set of lines can be defined if a DSS is considered. Let us denote by X the points of the DSS and by $\overline{X}$ the points of the DSS translated by the vector $(0, 1)$. The separating lines are the lines which are above the upper part of the convex hull (upper convex hull for short) of the points X and strictly below the lower part of the convex hull (lower convex hull for short) of the points $\overline{X}$ (see Fig. 1 for an illustration). Note that the strict constraint on the lower convex hull makes this definition slightly different from the classical definition in computational geometry. However, geometrically speaking, the

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