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Scheduling under a non-reversible energy source: An application of piecewise linear bounding of non-linear demand/cost functions

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ABSTRACT

In this paper, we address a preemptive scheduling problem involving a non-reversible energy source. To the classical scheduling issue, additional information is added regarding the characteristics of the energy source used to satisfy the total power demand of tasks processed at each instant. Different non-reversible energy sources can have very different characteristics in terms of power range and energy demand/cost function also known as efficiency function. The objective is to identify the best combination between scheduling and energy resource utilization that minimizes the total energy cost of the project. The non-linear efficiency function used to compute energy costs is bounded from above and below by two piecewise-linear curves, yielding two instances of a scheduling problem with a piecewise-linear objective that can be solved separately. For the piecewise-linear scheduling problem, we show that the problem involving multiple non-reversible sources is equivalent to a single-source problem, the particular case of a linear function is polynomially solvable, and the case with a piecewise-linear function with two pieces is NP-hard. A pseudo-polynomial size time-indexed mixed-integer linear formulation of the problem and its Dantzig–Wolfe decomposition yielding an extended formulation are presented. A branch-and-price procedure is proposed to solve the extended formulation. The formulations are compared on a set of scheduling instances, considering artificial and real-world efficiency functions.

1. Introduction

Energy considerations are becoming paramount for real-world applications. Moreover optimization issues are at the core of many industrial systems. A rising combinatorial optimization challenge is then the integration of energy constraints in deterministic scheduling and resource allocation models. We consider in this paper a scheduling problem where the objective is to minimize the total energy cost of a set of preemptive tasks subject to time windows. Each task has a constant energy demand when it is in process. Under a discrete time model, the set of tasks that are in process at a given time period generates a demand for this period. We assume that there are one or several non-reversible energy sources that can be used to meet this demand, such that the part of the demand covered by a source is converted in cost via a non-linear function.

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For a given source, the conversion function represents physical, technological, performance and/or billing characteristics. Contrary to reversible energy sources such as batteries and supercapacitors, non-reversible energy sources such as fuel cells, electric grid and combustion engines can produce energy but are unable to recover it (at least during the considered scheduling horizon).

We first give two practical examples of such non-linear demand/cost conversion function. The first example is the inclining block rate for electricity tariff, which is popular in many countries including the U.S. [23]. A customer having an energy consumption of Q kWh in a certain time period, will pay a bill $B = P_1 \min(Q, K) + P_2 \max(Q - K, 0)$ on that period where K is a threshold up to which the rate is P_1 while consumption above the threshold is billed with another rate $P_2 > P_1$. In Fig. 1(a), the expression of B is a function of Q for a threshold of 4.6 kW and prices $P_1 = 9.767$€c and $P_2 = 16.0485$€c. The function is piecewise linear. Note that the consumption is always measured on a period basis (typically 15 min, see the case study in the foundry industry in [14]). For other types of energy sources, obtaining the energy required by a set of tasks during a certain period can require a physical conversion subject to energy loss. For example, a Fuel Cell follows the scheme: *source* → *converter* → *usable energy* and the "energy cost" refers to the energy consumed from the source. In Fig. 1(b), the usable energy (demand) is expressed as a non-linear function of the consumed energy (cost) for a typical fuel cell used in hybrid electric vehicles [12,13]. Note that these functions are mostly polynomials.

This work is part of an ongoing effort aiming at solving explicitly and in an integrated fashion energy resource allocation problems and energy-consuming activity scheduling problems with such non-linear energy costs, yielding the following contributions.

First, together with a recent contribution [20], this paper introduces a novel and efficient methodology for the integration of energy characteristics in combinatorial optimization problems via piecewise-linear lower and upper bounding of the non-linear energy conversion functions. Second, we show that the preemptive scheduling problem with piecewise-linear energy cost from multiple non-reversible sources, can be transformed into an equivalent single-source problem. Third, we exhibit a polynomially-solvable case of the problem and we show its NP-hardness in general. Fourth, we provide mixed-integer linear programming (MILP) formulations, among which is an extended formulation. A branch-and-price procedure is proposed to solve the extended formulation. Computational experiments on a set of scheduling instances, considering artificial and real-world efficiency functions show the efficiency of our approach.

The remainder of the paper is organized as follows. Section 2 presents a review of relevant literature while Section 3 focusses on the piecewise-linear bounding framework. The definition, a first MILP formulation and the complexity analysis of the problem are provided in Section 4. Section 5 proposes an extended formulation resulting from the Dantzig–Wolfe decomposition of the first formulation. A branch-and-price procedure is described in Section 6. Finally, a computational evaluation of the propositions on scheduling instances with realistic efficiency functions is provided in Section 7, before stating the Conclusions.

2. Literature review

Over the years the stakes of resource allocation problems and production scheduling applications have evolved towards a more responsible management of resources. In particular new models in production scheduling were considered, where the energy demand can be modulated, mainly to avoid peaks of electrical consumption. A state-of-the-art review of energy concerns in production scheduling and a method for minimizing total energy cost on a single machine scheduling problem can be found in [7]. In this paper, as well as in many other related studies, the energy costs fluctuate over time. They depend on the market and on the processing state of the machine but are seldom subject to non-linear variations depending on demand. For more complex job-shop or resource-constrained project scheduling problems, the inclining block rate for electricity tariff yielding the piecewise-linear function with two pieces presented in Fig. 1(a) was considered in [14]. A scheduling problem in a foundry was introduced and a hybrid constraint-programming/MILP approach was proposed. The problem was further studied in [6] but although the non-linear electricity costs were considered, other non-linearities coming from energy modulation were ignored, as explained below. A metal is melted in induction furnaces and the electrical power of the furnaces can be adjusted at any time to avoid exceeding a maximum prescribed power limit. The electrical power can be seen as a continuous function of time to be determined, with the constraint that it must lie within minimum and maximum power levels that must be satisfied for the melting operation. More precisely, let $P_i(t)$ be the power used by operation i at time t, then the considered energy constraints state that the total energy consumed equal to $\int_{t_i}^{f_i} P_i(t)dt$, where t_i and f_i are the start and end times of the melting operation i, has to be equal to the operation energy demand W_i. This ignores the efficiency functions of the furnace. For the same type of adjustable power problems, constraint propagation algorithms on the basis of a continuous setting were proposed in [5] but energy efficiency functions were also ignored. A related work has also been carried out by Kis [16] who established polyhedral results and proposed a branch-and-cut procedure for a discretized time problem with variable-intensity tasks. Besides time discretization, the problem does not involve efficiency functions. Recently, the constraint propagation algorithms were extended to linear efficiency functions in [19].

Other studies consider explicitly non-linear energy constraints and costs. In scheduling, several authors considered different variants of the problem where the resource (not necessarily an energetic resource) usage may vary continuously and such that the amount of resource required by a task may vary over time. Węglarz et al. [22] call this issue the processing rate *vs.* resource amount model. The processing rate of the activity is a continuous increasing function of the allotted resource amount at a time, which corresponds to the efficiency function (see also [1,2]). Providing a general framework for solving