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Employee workload balancing by graph partitioning

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ABSTRACT

We consider the problem of partitioning a region into connected areas assigned to administrative officers. The employee in charge of an area takes care of all the activities which involve the towns of that area. An activity requires the effort of a subset of towns, coordinated by the employee in charge. This implies from the employee a fixed basic workload, plus a variable workload proportional to the number of towns involved. If the subset of towns associated to an activity is divided among several areas, the fixed workload is required from each of the corresponding employees, thus leading to a duplication. The problem requires to minimize duplications while balancing the workload among the employees.

The *Homogeneous Areas Problem* (*HAP*) models this situation as the search for a suitable balanced partition of a vertex-weighted and subset-weighted undirected graph into connected components. We propose a multi-commodity flow formulation, reduction procedures, a Tabu Search and a Very Large Scale Neighbourhood Search algorithm for the problem. We provide computational results for random instances and for two real-world instances, i. e. the provinces of Milan and Monza.

1. Introduction

Italy has adopted a federal and decentralized model of state administration. Town councils, in particular, are in charge of managing a large amount of matters involving their own territory. Several of these matters, however, involve different towns at the same time, and therefore can be managed efficiently only with a certain deal of coordination. The role to coordinate them is played by the *province*, an intermediate level of government between the towns and the central government. Each province periodically writes a Land Coordination Plan (*Piano Territoriale di Coordinamento*), which builds a framework for the overall policy of cooperation among the towns, and defines the procedures to implement coordinated actions. Then, the province works in joint with the single town administrations, providing them information and supporting their interactions on the matters of common interest.

Some provinces are rather large and involved in hundreds of different activities. This poses a problem of work organization, namely:

> how to achieve an effective interaction between the personnel of the towns and the experts of specific fields who work at the province.

The solution adopted by the province of Milan, which includes 134 towns, is to create a "customer care" layer of about 20 employees, with the task to support the employees of the towns. For the sake of efficiency, a certain degree of specialization

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in this layer is desirable, in order to limit the expertise required from each employee. To this purpose, the province is partitioned into "homogeneous areas", whose towns share a large number of common activities, different from the ones of the other areas. Then, a team of employees is assigned to each area and specifically trained on the corresponding activities. A by-product of this structure is to improve cooperation by means of personal relationships, since each team becomes a friendly and competent reference for the towns of the associated area. The workload imposed on the single teams should be similar for the sake of equity and is limited by the number of working hours available for the coordination activities. In general, this partition implies some redundancies, because a few activities involve towns from different areas, whose teams have to gain the same expertise and to perform the same operations. Such redundancies should be minimized. This gives rise to a special graph partitioning problem with a nonlinear objective function and additional side constraints which, to the best of our knowledge, has not yet been taken into account in the huge literature on graph partitioning problems. The solution provides a reference structure for the actual workflow organization, which of course should also take into account local agreements, as well as historical, social and political issues.

Section 2 describes the practical problem in detail and provides a formal definition. The following section discusses the computational complexity of the model, which turns out to be strongly $\mathcal{NP}$-hard. Section 4 presents the *HAP* in the general framework of graph partitioning, illustrating its specific features and discussing in detail the differences with respect to other related models. Section 5 proposes a Mixed Integer Programming (*MIP*) formulation and some propositions which allow to strengthen it. Sections 6.1 *and* 6.2 introduce a Tabu Search (*TS*) and a Very Large Scale Neighbourhood (*VLSN*) algorithm, which will be finally combined in a single algorithm. Section 7 discusses the results obtained by the two heuristics and by a commercial Mixed Integer Programming (*MIP*) solver applied to the mathematical formulation.

2. The homogeneous areas problem

In Italy, provinces play a coordination role: they support all local activities involving more than one town. Examples of such common activities are the management of some parks, the maintenance of some roads, the management of incentives to locally relevant economic sectors, the organization of cultural happenings, and so on. Experts on the technical and legislative issues related to these specific fields work for the different departments of the provincial administration. For each given activity, the employees of town administrations must refer to these experts and interact with the employees of the other towns involved. However, for many small towns with few officers, even referring to the right department or expert might be a complex, time-expensive task.

To provide a simpler and more effective structure from the point of view of the towns, the province administration offers a sort of "help desk", or friendly interface, under the form of a team of employees. This team supports the town employees on any matter of interest, addresses them to the experts in the right departments, keeps track of their needs, organizes meetings, and so on. This is also a way to improve cooperation by means of personal relationships between the employees of the towns and those of the province. In small provinces, a single team is sufficient. When the number of towns and activities grows larger, however, it is no longer possible to guarantee that all employees of the team have the necessary expertise on all the activities involving the province. Larger provinces, therefore, are partitioned into areas so that the team supporting each area can specialize on the corresponding activities. A desirable partitioning should group into the same area towns involved in the same activities and should disperse among different areas towns involved in different activities. In short, it should yield "homogeneous areas". Moreover, each area should correspond to a connected subset of towns, both because this makes it simpler to organize meetings for the town employees of the area and because disconnected areas are considered historically, psychologically, socially and politically "unacceptable". While this complicates the organization of the province, the opposite approach (requiring the towns to refer to different experts for each activity) would be less efficient for the single towns, which are much more numerous and have smaller human resources.

The size of the teams cannot be too large, because the members of each team should be interchangeable, and it cannot be too small, to avoid that vacations and illnesses might deprive a whole area of support. Two to four officers are considered a reasonable size. Since the total workload cannot exceed the available number of work hours, the size of the teams and the total workload imply a small range of reasonable values for the number of teams. Moreover, for equity reasons, the workload should also be distributed fairly among the teams.

The workload for each activity derives from two groups of elementary tasks: the former is devoted to the activity itself (keeping up-to-date with the legislation, organizing meetings, sending standard letters, etc ...), the latter is repeated for each town singularly (paying visits, answering questions, etc ...). An estimate of the workload can be made taking into account both groups of tasks.

Due to the connection and capacity constraints, as well as the heterogeneous nature of the activities, it might be impossible to keep all towns involved in the same activity in a single area, so that two or more teams might be forced to replicate the first group of elementary tasks. This is a source of redundancy and inefficiency, which should be minimized. Therefore, we are looking for a partition of the towns into connected areas with a balanced workload, such that the amount of replicated tasks is as little as possible.

A combinatorial definition. The *Homogeneous Areas Problem* (*HAP*) can be formulated as follows. Let $G = (V, E)$ be an undirected graph whose vertices stand for towns, and whose edges stand for pairs of neighbour towns. Let A denote the set of activities and $A_v \subseteq A$ the subset of activities involving each town $v \in V$. Conversely, let $V_a = \{v \in V : a \in A_v\}$ be the set of towns involved in activity a.