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Approximability of the two-stage stochastic knapsack problem with discretely distributed weights

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ABSTRACT

In this paper the two-stage knapsack problem with random weights is studied under the aspect of approximability. We assume finite probability distributions for the weights and show that, unless P = NP, the so obtained problem cannot be approximated in polynomial time within a better ratio than $K^{-1/2}$ (where K is the number of second-stage scenarios). We further study the special cases where in the second stage items can only be added or only be removed, but not both. Positive approximation results are given for three particular cases, namely linearly dependent first- and second-stage rewards, the polynomial scenario model and the case where the number of scenarios is assumed to be a constant. To the best of our knowledge, this is the first study of a two-stage knapsack problem under the aspect of approximability and the first time a non-approximability result has been proven for a stochastic knapsack problem of any kind.

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1. Introduction

The knapsack problem is a widely studied combinatorial optimization problem. Special interest arises from numerous real life applications for example in logistics, network optimization and scheduling. The basic problem consists in choosing a subset out of a given set of items such that the total weight (or size) of the subset does not exceed a given limit (the capacity of the knapsack) and the total reward of the subset is maximized (for a survey on (deterministic) knapsack problems see the book by Kellerer et al. [18]).

However, most real life problems are non-deterministic in the sense that some of the parameters are not (exactly) known at the moment when the decision of which items to choose has to be made. For example, the available capacity might be unknown due to delays in previous jobs or the item rewards might depend on market fluctuations [4,15,24]. One possibility to model and solve an optimization problem in the presence of uncertainty is to formulate it as a stochastic programming problem.

In this paper we study a stochastic knapsack problem where the item weights are random. Several variants of stochastic knapsack problems with random item weights (or sizes) have been studied so far and the interest seems still to increase. Among the publications recently released you can find papers on the simple-recourse knapsack problem [23], the chance-constrained knapsack problem [13], knapsack problems with recourse [11] as well as dynamic settings of the stochastic knapsack problem [2].

The problem studied in this work has a two-stage formulation. Two-stage optimization models (also known as optimization models with recourse) are important tools in decision making with uncertain parameters as in many cases a corrective decision can or even has to be made once the exact parameters are known. Therefore, they have been extensively studied since they were first introduced in the literature by Dantzig in 1955 (see [5]). For our stochastic knapsack model

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we assume that in the first stage, while the item weights are still unknown, a pre-decision of which items to include in the knapsack can be made. This decision can be corrected once all the item weights have come to be known. More precisely, items can be removed at a certain cost and/or additional items added, which naturally yields a smaller reward than in the first stage. We call the resulting problem the Two-Stage Knapsack problem with random weights (*TSKP*).

One can imagine various problems that could be modeled as *TSKP*s. In fact, this is true for any problem that in its deterministic form can be modeled as a knapsack problem and where one can think of cases where the weights are uncertain and thus a short-term correction of the decision might be needed once the weights are known for certain. For instance, take the example where the beds of a hotel complex or the seats of an airplane have to be filled, without knowing if there might be later cancellations or if an overbooking will lead to an excess of the capacity (see also Section 2.1 where the former example will be given more in detail). In logistics one might have to schedule a fleet without knowing the exact sizes of the goods to be transported: If, in the end, there is some spare space, this might be filled with supplementary items on short notice. In case of an overload, a penalty might have to be paid to the un or not fully served customers.

In this study of the *TSKP* we assume the random weight vector to be discretely distributed, i.e., to only admit a finite number of realizations with non-zero probability. In fact, Kleywegt et al. [20] have shown that a stochastic combinatorial optimization problem can, under some mild assumptions, be approximated to any desired precision by replacing the underlying distribution by a finite random sample.

It is well known that in the case of finite weight distributions the *TSKP* can be equivalently reformulated as a deterministic linear combinatorial programming problem (see e.g. [11]). However, it has been shown that two-stage problems with discrete distributions on (some of) the parameters are in general #*P*-hard (even in the case of continuous decision variables, see [8]). Moreover, the number of constraints and binary decision variables in the deterministic reformulation grows with the number of scenarios and is thus typically very large, or even exponential in the number of items (e.g., if we assume the random weights to be independently distributed). Solving the problem to optimality is thus only possible in very restricted cases. The aim of this paper is therefore to study the *TSKP* under the aspect of approximability. We show that, unless $\mathcal{P} = \mathcal{NP}$, the *TSKP* cannot be approximated within $K^{-\frac{1}{2}+\epsilon}$ in polynomial time, where K is the number of scenarios and $\epsilon > 0$. This is remarkable insofar, as the deterministic knapsack problem admits a very simple $\frac{1}{2}$-approximation algorithm as well as a Fully Polynomial Time Approximation Scheme (*FPTAS*). To obtain this non-approximability result we first show a key property of the *TSKP*: The solution to any instance of the *TSKP* can be obtained by solving an instance of a two-stage knapsack problem where, in the second stage, items can only be added (here called *AddTSKP*). The inverse is also true. This leads us to showing the mentioned key result for the *TSKP* by reducing the well studied multiply-constrained knapsack problem with uniform capacities to the *AddTSKP*.

We also show that the deterministic reformulation of the *AddTSKP* is in turn a multiply-constrained knapsack problem. Note however that, even if we assume the number of second-stage scenarios to be a constant, the deterministic reformulation of the *AddTSKP* has a number of constraints that is linear in the number of items and can thus not be assumed to be a constant. To the best of our knowledge, applying any so far published approximation algorithm for the multiply-constrained knapsack problem (or its basic idea) directly to the *AddTSKP* does not yield a polynomial time algorithm for exactly this reason. Instead, we propose an approximation algorithm with approximation ratio $\frac{1}{2} - \epsilon$ ($\epsilon \in (0, \frac{1}{2})$) where one has to approximately solve both a multiply-constrained knapsack problem and K (normal) knapsack problems. Moreover, we prove simple approximation algorithms for two more special cases of the *TSKP* (namely linearly dependent first- and second-stage rewards and the polynomial scenario model).

In the last decade, several papers have been published that treat the question of approximability of combinatorial two-stage problems. Some of these results are positive in the sense that the stochastic version of the problem is not much harder to solve than its deterministic counterpart [25,26]. For other problems, the introduction of stochasticity increases the problem's complexity significantly (see [7,10,17,21]) which can be due to one of the two following circumstances: In some cases the fact that we have at least two different possible second-stage scenarios is the reason for the increased complexity of the two-stage problem. In these cases the problem reduces to the deterministic counterpart in case the number of second-stage scenarios is 1. The *TSKP* clearly falls in this category of problems. In other cases the combinatorial structure of the problem changes completely when introducing a second stage. These problems can be characterized by the property that even in the case of only one possible second-stage scenario the problem stays harder to solve or approximate than its deterministic counterpart. For more information on approximation of two-stage optimization problems see the surveys by Swamy and Shmoys [28], Stougie and van der Vlerk [27] or Immorlica et al. [16].

The *TSKP* has, for instance, not been studied under the aspect of approximability. Moreover, to the best of our knowledge, we present in this paper the first non-approximability result for a stochastic knapsack problem of any kind.

Dean et al. [6] studied a stochastic knapsack problem where the weight of an item is not known until the item is placed in the knapsack. The authors do not make any assumption about the underlying probability distribution of the random parameters. The aim of their work is "to design a solution policy for sequentially inserting items until the capacity is eventually exceeded". They compare non-adaptive with adaptive policies (where in the latter case the choice of which item to insert next is made with respect to the already added items). The authors propose constant-factor approximation algorithms for both the adaptive and non-adaptive case. Their results have been recently improved (see [2]). Moreover, a new but much related problem has been studied where the knapsacks capacity is allowed to be extended by an arbitrary small amount. While in [2,6] items, once they have been inserted cannot be prematurely canceled any more, this is explicitly allowed in

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