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On the extremal values of the eccentric distance sum of trees

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Let G be a simple connected graph. The eccentric distance sum (EDS) of G is defined as $\xi^d(G) = \sum_{v \in V} \varepsilon_G(v) D_G(v)$, where $\varepsilon_G(v)$ is the eccentricity of the vertex v and $D_G(v) = \sum_{u \in V} d_G(u, v)$ is the sum of all distances from the vertex v. In this paper, the trees having the maximal EDS among n-vertex trees with maximum degree Δ and among those with domination number 3 are characterized. The trees having the maximal or minimal EDS among n-vertex trees with independence number α and the trees having the maximal EDS among n-vertex trees with matching number m are also determined.

1. Introduction

In this paper, all graphs $G = (V, E)$ are finite, simple and undirected. For $x \in V$, $N_G(x)$ (or $N(x)$) is the set of vertices adjacent to x, and the *degree* of x, denoted by $d_G(x)$ (or $d(x)$) is $|N_G(x)|$ (or $|N(x)|$). We call u a *leaf* if $d(u) = 1$. We use $\Delta(G)$ and $\delta(G)$(or Δ and δ) to denote the *maximum degree* and *minimum degree* of G, respectively. A vertex of degree k is called a k-*vertex*. If $uv \in E$, we call u (or v) a *neighbor* of v (or u). We call v a *pendent neighbor* of u if $uv \in E$ and $d(v) = 1$. For $u, v \in V$, the *distance* $d_G(u, v)$ is defined as the length of the shortest path between u and v in G. $D_G(v)$ denotes the sum of all distances from v. The *eccentricity* $\varepsilon(v)$ of a vertex v is the maximum distance from v to any other vertex. The *centre* of a graph is a vertex of minimum eccentricity. For $W \subseteq V$, $G - W$ denotes the graph obtained from G by deleting the vertices in W together with their incident edges. If $W = \{w\}$, we just write $G - w$ for $G - \{w\}$. If $U \subseteq V$, then $G[U]$ denotes the graph on U whose edges are precisely the edges of G with both ends in U. Let S_n, P_n and K_n be a star, a path and a complete graph on n vertices respectively. We use $l(P)$ to denote the length of a path P. For a real number x, we use $\lfloor x \rfloor$ to denote the greater integer no greater than x and use $\lceil x \rceil$ to denote the least integer no less than x.

A subset S of V is called a *dominating set* of G if for every vertex $v \in V \setminus S$, there exists a vertex $u \in S$ such that v is adjacent to u. The *domination number* of G, denoted by $\gamma(G)$, is defined as the minimum cardinality of dominating sets of G. For a connected graph G of order n, Ore [20] obtained that $\gamma(G) \leq \frac{n}{2}$.

A subset M of $E(G)$ is called a *matching* of G if no two edges are adjacent in G. The *matching number* of G, denoted by $\alpha'(G)$, is defined as the maximum cardinality of matching sets of G.

A subset S of $V(G)$ is called an *independent set* of G if no two vertices from S are adjacent in G. The *independence number* of G, denoted by $\alpha(G)$, is defined as the maximum cardinality of independent sets of G. It is known that for a bipartite graph G of order n and with $\delta > 0$, $\alpha'(G) + \alpha(G) = n$. All trees are bipartite, hence the equation holds for the theorems in Section 5.

A single number that can be used to characterize some property of the graph of a molecule is called a topological index. The topological index is a graph theoretic property that is preserved by isomorphism. The chemical information derived through the topological index has been found useful in chemical documentation, isomer discrimination, structure–property

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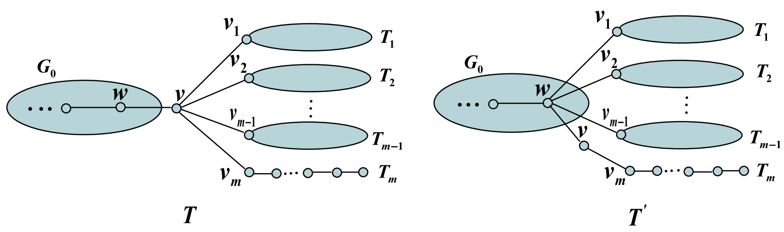

Fig. 1. ρ transformation.

correlations, etc. [1]. The properties of many topological indices such as the Wiener index [3], the degree distance index, the eccentric connectivity index [21,4,9,15,16,19], the eccentric distance sum [10] and so on are established and used as molecular descriptors [11,12]. Some results on the domination number can be found in [2,5–7,22,23].

The graph invariant-eccentric distance sum (EDS) was introduced by Gupta, Singh and Madan [10], which was defined as

$$\xi^d(G) = \sum_{v \in V} \varepsilon_G(v) D_G(v).$$

The eccentric distance sum can also be defined as

$$\xi^d(G) = \sum_{u,v \in V} (\varepsilon_G(u) + \varepsilon_G(v)) d_G(u, v).$$

Many researchers have studied the eccentric distance sum of trees. Yu, Feng and Ilić [24] characterized the trees with the minimal EDS among the n-vertex trees of a given diameter. Li, Zhang, Yu, Feng [18] identified the trees with the minimal and second minimal eccentric distance sums among the n-vertex trees with matching number q and characterized the extremal tree with the second minimal eccentric distance sum among the n-vertex trees of a given diameter. They also determined the trees with the third and fourth minimal eccentric distance sums among the n-vertex trees. Geng, Li and Zhang [8] characterize the trees with the minimal EDS among n-vertex trees with domination number γ, and determine the trees with the maximal EDS among n-vertex trees with domination number γ satisfying $n = k\gamma$, where $k = 2, 3, \frac{n}{2}$. They also identify the trees with the minimal and the maximal EDS among the n-vertex trees with k leaves. Other results about the EDS of graphs can also be found in [13,14,17].

In this paper, we continue to study the eccentric distance sum of trees. The trees having the maximal EDS among n-vertex trees with maximum degree Δ and among those with domination number 3 are characterized. The trees having the maximal or minimal EDS among n-vertex trees with independence number α and the trees having the maximal EDS among n-vertex trees with matching number m are also determined.

2. Preliminaries

In this paper, we will use two graph transformations A_1 and A_2 posed in [14] and [8], respectively.

A_1: Let T be a tree of order $n > 3$ and $e = uv$ be a nonpendent edge. Suppose that $T - e = T_1 \cup T_2$ with $u \in V(T_1)$ and $v \in V(T_2)$. A new tree T_0 is obtained by identifying the vertex u of T_1 with vertex v of T_2 and attaching a leaf to the $u(= v)$. T_0 is said to be obtained by running an edge-growing transformation of T (on edge $e = uv$), or e.g.t of T (on edge $e = uv$) for short.

Let T be a tree and uv be a pendent edge with $d_T(v) = 1$ and $d_T(u) \geq 3$. Suppose $uw \in E(T)$ and $w \neq v$. Let $T_0 = T - \{uw\} + \{vw\}$. Then T_0 is said to be obtained by *running converse of e.g.t* of T on uw.

Lemma 2.1 ([14]). *Let T be a tree of order $n > 3$ and $e = uv$ be a nonpendent edge of T. If T_0 is a tree obtained from T by running one step of e.g.t (on edge $e = uv$), then we have $\xi^d(T_0) < \xi^d(T)$.*

A_2: Let T be an arbitrary tree rooted at a center vertex and let v be a vertex of degree $m + 1 (m \geq 2)$. Suppose that w is adjacent to v with $\varepsilon_T(v) \geq \varepsilon_T(w)$ and that $T_1, T_2, \ldots, T_m$ are subtrees under v with root vertices $v_1, v_2, \ldots, v_m$ where $N_T(v) = \{w, v_1, v_2, \ldots, v_{m-1}, v_m\}$ and T_m is actually a path. Let $T' = T - \{vv_1, vv_2, \ldots, vv_{m-1}\} + \{wv_1, wv_2, \ldots, wv_{m-1}\}$. We say that T' is a ρ *transformation* of T and denote it by $T' = \rho(T, v)$. See Fig. 1.

Lemma 2.2 ([14]). *Let T' be a ρ transformation of T defined as above, one has $\xi^d(T) \geq \xi^d(T')$. The equality holds if and only if $\varepsilon_T(v) = \varepsilon_T(w)$ and $T[S]$ is one of the longest paths in T, where $S = V(G_0) \cup V(T_m) \cup \{v\}$ and G_0 is the graph obtained from T (or T') by deleting $V(T_1) \cup V(T_2) \cup \cdots \cup V(T_m) \cup \{v\}$.*

A *spider* is a tree with at most one vertex of degree more than 2, called the *hub* of the spider (if there is no vertex of degree more than two, then any vertex can be the hub). A *leg* of a spider is a path from the hub to one of its leaves. Let $S(a_1, a_2, \ldots, a_k)$ be a spider with k legs $P^1, P^2, \ldots, P^k$ for which the lengths $l(P^i) = a_i\, (i = 1, 2, \ldots, k)$. It holds that