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Note

Topological properties on the diameters of the integer simplex

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ABSTRACT

Wide diameter $d_\omega(G)$ and fault-diameter $D_\omega(G)$ of an interconnection network G have been recently studied by many authors. We determine the wide diameter and fault-diameter of the integer simplex T_m^n. Note that $d_1(T_m^n) = D_1(T_m^n) = d(T_m^n)$, where $d(T_m^n)$ is the diameter of T_m^n. We prove that $d_\omega(T_m^n) = D_\omega(T_m^n) = d(T_m^n) + 1$ when $2 \le \omega \le n$. Since a triangular pyramid TP_L is T_L^3, we have $d_\omega(TP_L) = D_\omega(TP_L) = d(TP_L) + 1$ when $2 \le \omega \le 3$.
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1. Introduction

An interconnection network is conveniently represented by an undirected graph. The vertices(edges) of the graph represent the nodes(links) of the network. As a topology for an interconnection network of a multiprocessor system, the triangular pyramid (tripy for short), is proposed by Razavi and Sarbazi-Azad in [14]. Some basic properties such as Hamiltonian-connectivity, pancyclicity and a routing algorithm were investigated in the paper. We studied other properties such as symmetry, connectivity and fault-tolerant vertex-pancyclicity in [13].

Reliability and efficiency are important criteria in the design of interconnection networks. In graph theory, the study of fault-tolerance and transmission delay of networks, wide diameter and fault-diameter are two very important parameters and have been studied by many researchers. The diameters of Cartesian product graphs were studied in [1–3,6,7,17]. The parameters of some well-known networks such as hypercube, crossed cube, etc., were studied in [4,5,9,11,12,15,18].

Wide diameter of a graph, which combines connectivity with diameter, is a parameter that measures simultaneously the fault-tolerance and efficiency of parallel processing computer networks. Let u and v be two vertices of a graph G. The distance between u and v, denoted by $d_G(u, v)$, is the length of a shortest path between them. The diameter of G, denoted by $d(G)$, is the maximum distance between any two vertices. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial network. We say that G is k-connected for any $0 < k \le \kappa(G)$. According to Menger's theorem, there are k disjoint paths between any two vertices in a k-connected network. Let G be a k-connected graph with $1 \le \omega \le k$. The ω-wide diameter $d_\omega(G)$ of G is the minimum ℓ such that there exist ω internally vertex disjoint paths of length at most ℓ from u to v for any two vertices u and v. Throughout this paper, "disjoint paths" always means "internally vertex disjoint paths". In particular, $d_1(G)$ is just the diameter $d(G)$ of G. It is easy to see that

$$d(G) = d_1(G) \le d_2(G) \le \cdots \le d_{k-1}(G) \le d_k(G).$$

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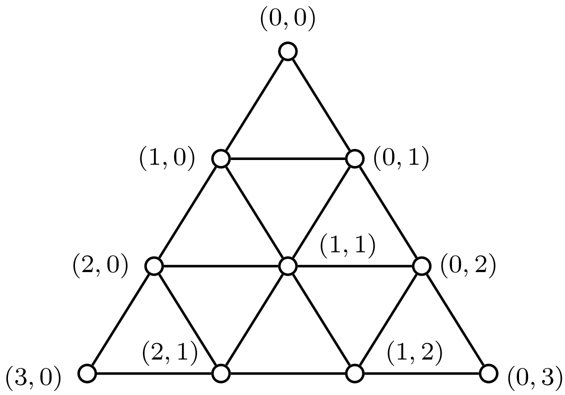

Fig. 1. The T_3.

Failures are inevitable when a network is put in use. Therefore, it is significant to consider faulty networks. The fault-diameter can be used to estimate the effect on the diameter when faults occur. A small fault-diameter is desirable because the delay would be shorter when some nodes fail. The fault-diameter with faulty vertices was first studied by the authors in [10]. The $(\omega - 1)$-fault-diameter of a graph G is defined as

$$D_\omega(G) = \max\{d(G - F) : F \subseteq V(G), |F| < \omega\}$$

where $G - F$ denotes the subgraph induced by $V(G) - F$. Note that $D_\omega(G) < \infty$ if and only if G is ω-connected. It is also clear that

$$d(G) = D_1(G) \le D_2(G) \le \cdots \le D_{k-1}(G) \le D_k(G).$$

It is well known (see [12]) that for any k-connected graph G and any integer ω, $1 \le \omega \le k$, we have $D_\omega(G) \le d_\omega(G)$, where the equality holds for some well-known networks. However, it is difficult to determine the wide diameter or fault-diameter of the tripy according to its definition in [14]. Fortunately, we find that the tripy TP_L is a special integer simplex, and we determine the wide diameter and fault-diameter of the integer simplex. The two kinds of diameters of the tripy are deduced from the results of the integer simplex.

The rest of this paper is organized as follows. We give some definitions and notations in Section 2. The main results are given in Section 3.

2. The integer simplex T_m^n

For graph-theoretical terminology and notation not defined here, we follow [16]. We first restate the definitions of triangular mesh and tripy originally proposed by Razavi and Sarbazi-Azad for completeness.

Definition 1. A radix-m triangular mesh network, denoted by T_m, consists of a set of vertices $V(T_m) = \{(x, y)|0 \le x+y \le m\}$ where any two vertices (x_1, y_1) and (x_2, y_2) are connected by an edge if and only if $|x_1 - x_2| + |y_1 - y_2| = 1$, or $x_2 = x_1 + 1$ and $y_2 = y_1 - 1$, or $x_2 = x_1 - 1$ and $y_2 = y_1 + 1$.

Fig. 1 shows a T_3 network.

A tripy is a hierarchy structure based on triangular meshes.

Definition 2. An L-level tripy, denoted by TP_L, consists of a set of vertices $V(TP_L) = \{(k, (x, y))|0 \le k \le L, 0 \le x + y \le k\}$. Vertex $(k, (x, y)) \in V(TP_L)$ is said to be a vertex at level k with the coordinate (x, y). The vertices at level k form a network of T_k. Vertex $(k, (x, y))$ is also connected to vertices (x, y), $(x + 1, y)$, and $(x, y + 1)$, in level $k + 1$, as child vertices, and to vertex $(x - 1, y)$ if $x > 0$, vertex $(x, y - 1)$ if $y > 0$, and vertex (x, y) if $x + y < k$, as parents in level $k - 1$.

Definition 3. The integer simplex with dimension n and side-length m is the graph T_m^n whose vertices are the nonnegative integer $(n + 1)$-tuples summing to m, with two vertices adjacent when they differ by 1 in two places and are equal in all other places.

In other words,

$$V(T_m^n) = \{v_n \cdots v_1 v_0 \mid \Sigma_{i=0}^n v_i = m,\ v_i\ (i \in \{0, 1, \ldots, n\}) \text{ is a non-negative integer}\}.$$

Two vertices $u = u_n \cdots u_1 u_0$ and $v = v_n \cdots v_1 v_0$ are adjacent if and only if $u_i = v_i - 1$, $u_j = v_j + 1$ and $u_k = v_k$ where $k \in \{0, 1, \ldots, n\} \setminus \{i, j\}$.

However, both the triangular mesh and the tripy are special kinds of integer simplex. Let σ_1 be a mapping from $V(T_m)$ to $V(T_m^2)$ defined by $\sigma_1((x, y)) = (m - (x + y), x, y)$, for any vertex $(x, y) \in V(T_m)$. Then, σ_1 is an isomorphism from T_m

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