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On the transportation problem with market choice

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ABSTRACT

We study a variant of the classical transportation problem in which suppliers with limited capacities have a choice of which demands (markets) to satisfy. We refer to this problem as the transportation problem with market choice (TPMC). While the classical transportation problem is known to be strongly polynomial-time solvable, we show that its market choice counterpart is strongly NP-complete. For the special case when all potential demands are no greater than two, we show that the problem reduces in polynomial time to minimum weight perfect matching in a general graph, and thus can be solved in polynomial time. We give valid inequalities and coefficient update schemes for general mixed-integer sets that are substructures of TPMC. Finally, we give conditions under which these inequalities define facets, and report our preliminary computational experiments with using them in a branch-and-cut algorithm.

1. Introduction

We consider a variant of the classical transportation problem in which suppliers with limited capacities have a choice of which demands (markets) to satisfy. In this problem, if a market is selected, then its demand must be satisfied fully through shipments from the suppliers. If a market is rejected, then the corresponding potential revenue is lost. The objective is to minimize the total cost of shipping and lost revenues. We refer to this problem as the transportation problem with market choice (TPMC).

More formally, we are given a set of supply and demand nodes that form a bipartite graph $G(V_1 \cup V_2, E)$. The nodes in set V_1 represent the supply nodes, where for $i \in V_1$, $s_i \in \mathbb{N}$ represents the capacity of supplier i. The nodes in set V_2 represent the potential markets, where for $j \in V_2$, $d_j \in \mathbb{N}$ represents the demand of market j. The edges between supply and demand nodes have weights that represent shipping costs w_{ij}, where $(i,j) \in E$. For each $j \in V_2$, r_j is the revenue lost if the market j is rejected. For a given vector of parameters γ_j for $j \in S$ and $S' \subseteq S$, we let $\gamma(S') := \sum_{j \in S'} \gamma_j$, throughout the paper.

Let x_{ij} be the amount of demand of market j satisfied by supplier i for $(i,j) \in E$, and let z_j be an indicator variable taking a value 1 if market j is rejected and 0 otherwise. A mixed-integer programming (MIP) formulation of the problem is given where the objective is to minimize the transportation costs and the lost revenues due to unchosen markets:

$$\min \sum_{(i,j)\in E} w_{ij}x_{ij} + \sum_{j \in V_2} r_j z_j \tag{1a}$$

$$\text{s.t.} \sum_{i:(i,j)\in E} x_{ij} = d_j(1 - z_j) \quad \forall j \in V_2 \tag{1b}$$

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$$\sum_{j:(i,j)\in E} x_{ij} \le s_i \quad \forall i \in V_1 \tag{1c}$$

$$z \in \{0,1\}^{|V_2|} \tag{1d}$$

$$x \in \mathbb{R}_+^{|E|}. \tag{1e}$$

We refer to problem description (1a)–(1e) as TPMC. The first set of constraints (1b) is the demand constraint. In TPMC either a demand for a market is fully satisfied or rejected altogether, which necessitates the introduction of the additional binary variables. The second set of constraints (1c) model the supply restrictions.

TPMC is closely related to the capacitated facility location (CFL) problem. In CFL, given a set of potential facilities $j \in V_2$ with capacities $\bar{d}_j$, $j \in V_2$ and customers $i \in V_1$ with demands $\bar{s}_i$, $i \in V_1$, we would like to determine which facilities to open so that the demand of all customers can be satisfied from shipments from the open facilities. A MIP formulation of CFL is

$$\sum_{i:(i,j)\in E} \bar{x}_{ij} \le \bar{d}_j \bar{z}_j \quad \forall j \in V_2 \tag{2a}$$

$$\sum_{j:(i,j)\in E} \bar{x}_{ij} = \bar{s}_i \quad \forall i \in V_1 \tag{2b}$$

$$\bar{z} \in \{0,1\}^{|V_2|} \tag{2c}$$

$$\bar{x} \in \mathbb{R}_+^{|E|}. \tag{2d}$$

Therefore one may view the CFL problem as a 'complement' of the TPMC problem where the constraints (1b) and (1c) of TPMC change signs in the constraints (2a) and (2b) in CFL respectively. While the CFL problem has been extensively studied with respect to its complexity, polyhedral structure, and approximability ([1,6] and references therein), TPMC is less understood.

Recently, approximation algorithms and heuristics have been proposed for various supply chain planning and logistics problems with market choice [9,16]. It is assumed that these problems are uncapacitated or that they have *soft* capacities. A two-stage approach is utilized in solving these classes of problems that admit a facility location formulation. In the first stage, the problem is to determine a subset of markets and reject the others. In the second stage, the goal is to minimize the production cost and lost revenues due to unselected markets. In particular, for the *uncapacitated* lot-sizing problem, the facility location formulation is used to model the market choice counterpart. It is shown that the LP relaxation solution can be rounded in a way that guarantees a constant factor approximation algorithm. However, this algorithm relies on scaling continuous variables up, so it does not immediately generalize to our problem with hard capacity constraints (1c). Van den Heuvel et al. [23] consider a maximization version of the same problem and show that no constant factor approximation algorithm exists for this version, unless P = NP. The authors also give several polynomially solvable special cases, and test heuristics for the general case.

The rest of the paper is organized as follows. In Section 2, we explore the complexity of TPMC. We show that while the classical transportation problem admits a strongly polynomial algorithm [14], its market choice counterpart is strongly NP-complete. We also identify a polynomially solvable case when the demands of all potential markets are no more than two. In Section 3, we present methods for constructing valid inequalities for mixed integer cover sets and mixed-integer knapsack sets with variable upper bound constraints, which appear as substructures of TPMC. We show that these methods are useful for generating valid inequalities for TPMC. We also study the strength of the proposed valid inequalities. Our preliminary computations, summarized in Section 4, show that there is a reduction in the end gap when our valid inequalities are incorporated to the branch-and-cut algorithm. However, we do not give an extensive computational study and the heuristic separation we use needs significant improvement.

2. Complexity

We first show that TPMC is strongly NP-hard in general.

Proposition 1. *The decision version of TPMC is NP-complete even when:*

1. $s_i = 1$ *for all* $i \in V_1$, $d_j = d \ge 3$ *for all* $j \in V_2$, $w_{ij} = 0$ *for all* $(i,j) \in E$ *and* $r_j = 1$ *for all* $j \in V_2$.
2. $|V_1| = 1$ *and* $w_{ij} = 0$ *for all* $(i,j) \in E$.

The proof for Proposition 1 Part 1 is similar to the proof of a related result presented in [20]. For completeness, we provide its proof and the proof of Part 2 in the Appendix. Because the reduction of Part 1 is from the Exact 3-Cover problem, which is strongly NP-complete [8], we conclude that TPMC is strongly NP-hard even for the case where all demands are equal to three. In contrast, Proposition 2 shows that TPMC is polynomially solvable when demands of all markets do not exceed two.

Proposition 2. *Suppose that* $d_j \le 2$ *for all* $j \in V_2$. *Then there exists a polynomial-time algorithm to solve TPMC.*

This result is proven by a polynomial time reduction to a minimum weight perfect matching problem on a general graph (provided in the Appendix). The key ideas of the reduction are based on those presented in [2]. This result can also be proven by a polynomial time reduction to the b-matching problem [7], see also Theorem 36.1 in [21].