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Lot sizing with minimum order quantity

Young Woong Park *, Diego Klabjan

Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, IL, USA

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ABSTRACT

We consider the single item lot sizing problem with minimum order quantity where each period has an additional constraint on minimum production quantity. We study special cases of the general problem from the algorithmic and mathematical programming perspective. We exhibit a polynomial case when capacity is constant and minimum order quantities are non-increasing in time. Linear programming extended formulations are provided for various cases with constant minimum order quantity.

1. Introduction

The single item economic lot sizing problem is to find the production lot sizes of one item over several periods with the minimum cost. In this paper, we consider the single item lot sizing problem with minimum order quantity (MOQ). MOQ is an order requirement imposing that the amount of the production has to be at least a certain quantity when the period has a positive production. The MOQ requirement can be a hard constraint if it is due to business requirements such as the product required to be shipped in containers or pallets. However, it can be used as an alternative to fixed ordering cost or setup cost, as both of them prevent small orders that cause high per unit fixed or setup cost [10]. Hence, the MOQ requirement is an alternative way to achieve economies of scales in production and transportation [20]. Musalem and Dekker [10] and Zhao and Katehakis [20] provide real world cases where MOQ is used.

Given T periods, demands $d_1, \dots, d_T$ must be satisfied by a sequence of production schedules, where the production level in period t must be at least l_t and no more than u_t, if it is positive, for all $t = 1, \dots, T$. For notational convenience, let $d_{i,j} = \sum_{t=i}^{j} d_t$ be the summation of the demand from period i to j for $1 \le i \le j \le T$. The feasibility set of the single item lot sizing problem with MOQ is a set $S(l_t, u_t) \subseteq \mathbb{R}^T$ defined as

$$S(l_t, u_t) = \left\{ x \in \mathbb{R}^T : \begin{array}{ll} \sum_{t=1}^{j} x_t \ge d_{1,j}, & j = 1, \dots, T \\ x_t \in \{0\} \cup [l_t, u_t], & t = 1, \dots, T \end{array} \right\},$$

where x_t is the production level in period t. When there is no MOQ, we denote the set as $S(\emptyset, u_t)$ and when the MOQ's are constant, we denote the set as $S(l, u_t)$. When there exists no upper bound, we denote the set as $S(l_t, \infty)$. Similarly, $S(l, u)$, $S(l, \infty)$, $S(\emptyset, u)$ are defined. In the literature, based on our convention, $S(\emptyset, u_t)$ and $S(\emptyset, u)$ are referred as capacitated lot

* Corresponding author. Tel.: +1 8474918470; fax: +1 8474918005.
E-mail addresses: ywpark@u.northwestern.edu (Y.W. Park), d-klabjan@northwestern.edu (D. Klabjan).

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Table 1
Polynomial cases with MOQ.

	Constant capacity	Uncapacitated
Constant MOQ	$S(l, u)$: Hellion et al. [6] Okhrin and Richter [11]	$S(l, \infty)$: Lee [7] Okhrin and Richter [12]
Non-constant MOQ	$S(l_t, u)$: Section 2 in this paper	$S(l_t, \infty)$: Li et al. [8]

sizing, and $S(\emptyset, \infty)$ is referred as uncapacitated lot sizing. Any feasibility set with a lower bound such as $S(l_t, u)$, $S(l, u)$, or $S(l_t, \infty)$ is referred as lot sizing with MOQ.

In our work, we consider time-dependent and constant MOQ with constant capacity on a finite time horizon. We present a polynomial algorithm for the single item lot sizing problem with non-constant and non-increasing MOQ with non-increasing linear costs. The algorithm is based on the well-known dynamic programming (DP) algorithm of Florian and Klein [5]. For each node of the shortest path network of the underlying DP, we solve the corresponding optimization problem greedily. For the lot sizing problem with constant MOQ, we present linear programming (LP) extended formulations for the single item lot sizing problem with constant MOQ and upper bound with non-increasing linear costs.

Since the seminal works of Manne [9] and Wagner and Whitin [19], the lot sizing problem has been extensively studied. In this review, we focus on the lot sizing problem such that demand is deterministic and order quantity is the only decision. To decrease the scope of the review further and to align it with the work presented herein, we consider

1. polynomial algorithms for the single item lot sizing problem with MOQ, and
2. polyhedral study and LP extended formulations for the lot sizing problem and related problems.

Since Anderson and Cheah [1] introduced the MOQ constraint for the multi-item lot sizing problem, there have been several studies on polynomial algorithms for the single item lot sizing problem with MOQ. Lee [7] provided the first polynomial algorithm for $S(l, \infty)$. Li et al. [8] exhibit a polynomial algorithm for $S(l_t, \infty)$ with non-increasing cost and MOQ. Okhrin and Richter [12] also studied an algorithm for $S(l, \infty)$. Okhrin and Richter [11] provided a polynomial algorithm for $S(l, u)$ with constant holding cost. Hellion et al. [6] also studied $S(l, u)$ with concave costs. Our polynomial case for $S(l_t, u)$ is different from the other polynomial cases as no previous study considered non-constant MOQ together with upper bounds. The polynomial cases for the lot sizing problem with MOQ are summarized in Table 1, considering only the different capacities and MOQ requirements.

Polyhedra and LP extended formulations for the lot sizing problem also have been studied in the literature. Pochet [13] studied valid inequalities and facets of $S(\emptyset, u)$. Pochet and Wolsey [15] gave a tight and compact reformulation for $S(\emptyset, u)$ in the presence of the Wagner–Whitin cost. Constantino [4] studied the polyhedron of a relaxation of $S(l, u)$. Van Vyve [18] provided LP extended formulations for $S(\emptyset, u)$ with backlogging. Anily et al. [3] provided an LP extended formulation for multi-item lot sizing where each item belongs to $S(\emptyset, u)$. Pochet and Wolsey [17] proposed a compact mixed integer programming reformulation whose LP relaxation solves $S(\emptyset, u_t)$ when capacities u_t's are non-decreasing over time. To the best of our knowledge, an LP extended formulation for the single item lot sizing problem with MOQ has not yet been studied. Recently, Angulo et al. [2] studied the semi-continuous inflow set of a single node of the type $S(l_t, \infty)$. They provided an LP extended formulation for the semi-continuous inflow set. Our work is distinguished from the work in [2], since $S(l, u)$ is very different from the set they considered.

Our contribution can be summarized as follows.

1. In addition to the cases that are already proved to be polynomial, we identify that $S(l_t, u)$ with non-increasing linear costs and non-increasing l_t's can be solved in polynomial time, by providing a polynomial algorithm for the first time in the literature.
2. We provide various LP extended formulations for $S(l, u)$ and $S(l, \infty)$ with non-increasing costs. The proposed formulations are the first LP extended formulations for the single item lot sizing problem with the presence of MOQ.

The rest of the paper is organized as follows. Section 2 provides a polynomial time algorithm for $S(l_t, u)$ with additional assumptions on the orders of the lower bounds and objective cost coefficients. Section 3 develops the LP extended formulations for $S(l, u)$ and $S(l, \infty)$ with non-increasing production and fixed costs. In Section 4, we present the computational experiments for the proposed algorithm and formulations.

2. Polynomial case

In this section, we show that $S(l_t, u)$ with a linear cost function can be solved in polynomial time if we additionally assume the following.

Non-increasing cost $\quad p_1 \geq p_2 \geq \cdots \geq p_T$
Non-increasing lower bound $\quad l_1 \geq l_2 \geq \cdots \geq l_T$

We also assume that the demand satisfies $d_t \leq u$ for all t. The reader is referred to the book by Pochet and Wolsey [16] for the justification of this assumption after an appropriate preprocessing of d_t's. The key idea of the preprocessing is that,