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The decycling number of generalized Petersen graphs ☆

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ABSTRACT

A subset $F \subset V(G)$ is called *a decycling set* if the subgraph $G - F$ is acyclic. The minimum cardinality of a decycling set is called *the decycling number* of G, which is proposed first by Beineke and Vandell (1997). We use $\nabla(P_{n,k})$ to denote the decycling number of the generalized Petersen graphs $P_{n,k}$. This paper proves that

$$\nabla(P_{n,k}) = \begin{cases} \left\lceil \dfrac{n+1}{2} \right\rceil, & \text{if } n \neq 2k, \\ \left\lceil \dfrac{k+1}{2} \right\rceil, & \text{if } n = 2k. \end{cases}$$

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1. Introduction

Let $G = (V, E)$ be a simple graph, with vertex set V and edge set E. A subset $F \subset V(G)$ is called *a decycling set* if the subgraph $G - F$ is acyclic, that is, if $G - F$ is a forest. The minimum cardinality of a decycling set is called *the decycling number* (or *feedback number*) of G, which is proposed first by Beineke and Vandell [3]. A decycling set of this cardinality is called a minimum decycling set (or *feedback set*).

Determining the decycling number of a graph G is equivalent to finding the greatest order of an induced forest of G proposed first by Erdős, Saks and Sós [4], since the sum of the two numbers equals the order of G. A review of several results and open problems on the decycling number was provided by Bau and Beineke [2].

In fact, the problem of finding the decycling number is *NP*-hard for graphs in general [7] (also see [6]). The best known approximation algorithm for this problem has approximation ratio 2 [1].

Determining the decycling number is quite difficult even for some elementary graphs. We refer the reader to an original research paper [3] for some results. Bounds on the decycling numbers have been established for some well-known graphs, such as hypercubes [5], star graphs [9], (n, k)-star graphs [10], distance graphs and circulant graphs [8].

In this paper, we consider a particular topology graph called a generalized Petersen graph. We use $\nabla(P_{n,k})$ to denote the decycling number of $P_{n,k}$, this paper proves that $\nabla(P_{n,k}) = \lceil \frac{n+1}{2} \rceil (n \neq 2k)$, $\nabla(P_{2k,k}) = \lceil \frac{k+1}{2} \rceil$. The proof of the result is in Section 3. In Section 2, we obtain a lower bound of decycling number of $P_{n,k}$.

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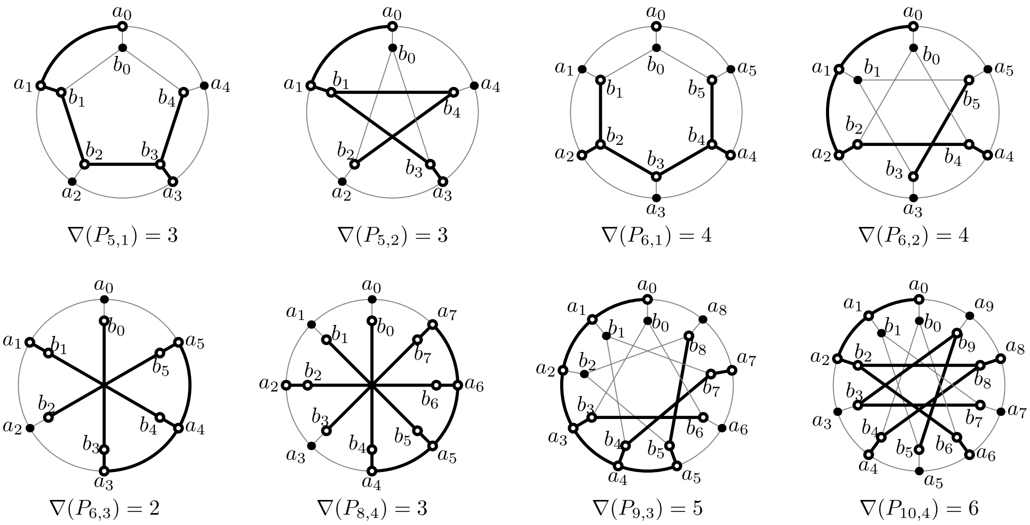

Fig. 2.1. The decycling sets of several $P_{n,k}$ graphs.

2. Lower bound of $\nabla(P_{n,k})$

The generalized Petersen graph $P_{n,k}$ is an important 3-regular graph on $2n$ vertices with $V(P_{n,k}) = \{a_i, b_i : 0 \le i \le n-1\}$ and $E(P_{n,k}) = \{a_ib_i, a_ia_{i+1}, b_ib_{i+k} : 0 \le i \le n-1, \text{ subscripts mod} n\}$.

Clearly, $P_{n,k} \cong P_{n,n-k}$. So we only need to consider the case $k \le \frac{n}{2}$. As we shall see, although the methods of proof are similar, the results in the two cases of $n = 2k$ and $n > 2k$ are somewhat different, and so we state the results separately until the conclusion.

We first restate a lower bound on the size of the decycling set in any graph, due to Beineke and Vandell [3].

Lemma 2.1. *If G is a graph with n vertices, m edges, and maximum degree $\Delta > 0$, then*

$$\nabla(G) \ge \left\lceil \frac{m-n+1}{\Delta - 1} \right\rceil.$$

Lemma 2.2. (a) $\nabla(P_{2k,k}) \ge \lceil \frac{k+1}{2} \rceil$. (b) *For* $k < \frac{n}{2}$, $\nabla(P_{n,k}) \ge \lceil \frac{n+1}{2} \rceil$.

Proof. The result follows at once from Lemma 2.1 and the observations that $P_{2k,k}$ has $4k$ vertices and $5k$ edges, and for $k < n$, $P_{n,k}$ has $2n$ vertices and $3n$ edges. □

In Fig. 2.1, we illustrate the decycling sets of several $P_{n,k}$ graphs with small n and k, where the vertices of decycling sets are solid. It is easy to check that all the decycling sets achieve lower bounds given by Lemma 2.2.

3. Decycling number of $P_{n,k}$

We now construct decycling sets that achieve these bounds. In order to simplify the proof, we first study cycles in $P_{n,k}$. Let $V_a = \{a_i | i = 0, 1, \ldots, n-1\}$ and $V_b = \{b_i | i = 0, 1, \ldots, n-1\}$, $G_a = G[V_a]$, $G_b = G[V_b]$. Let $d = \gcd(n, k)$.

Lemma 3.1. *The graph G_b is the union of d disjoint cycles with length $\frac{n}{d}$ and each cycle can be represented by* $Z_j = b_jb_{j+k}b_{j+2k} \cdots b_{j+(\frac{n}{d}-1)k}b_j$, *subscripts* mod$n$, $j = 0, 1, 2, \ldots, d-1$.

Proof. Clearly for each $j \in \{0, 1, 2, \ldots, d-1\}$, the sequence j, $(j+k)\text{mod}n$, $(j+2k)\text{mod}n, \ldots, (j+ik)\text{mod}n, \ldots$ will be periodic. If i is the least positive integer for which $(j+ik)\text{mod}n = j$, then we have $ik \equiv 0(\text{mod } n)$. Since $\gcd(n,k) = d$ and i is minimum, we have $i = \frac{n}{d}$ and that means a period of the sequence is $\frac{n}{d}$, which implies $(b_j, b_{j+k}, b_{j+2k}, \ldots, b_{j+ik}, \ldots, b_{j+n-k})$ is a cycle with length $\frac{n}{d}$. In order to prove that different values of j give disjoint cycles, we assume $(j_1 + i_1k)\text{mod}n = (j_2 + i_2k)\text{mod}n$, then $j_1 - j_2 + (i_1 - i_2)k \equiv 0(\text{mod } n)$, that is $j_1 - j_2 \equiv (i_2 - i_1)k(\text{mod } n)$. Since $d|n$, then we have $j_1 - j_2 \equiv (i_2 - i_1)k(\text{mod } d)$. Moreover $d|k$, then $j_1 - j_2 \equiv 0(\text{mod } d)$. As $j_1 < d, j_2 < d$, finally we have $j_1 = j_2$. Thus, d cycles of graph G_b with length $\frac{n}{d}$ are disjoint from each other and each cycle can be represented by $Z_j = b_jb_{j+k}b_{j+2k} \cdots b_{j+(\frac{n}{d}-1)k}b_j$, subscripts mod$n$, $j = 0, 1, 2, \ldots, d-1$. □

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