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On the connectivity properties and energy of Fibonomial graphs

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ABSTRACT

We introduce a new type of graph called a Fibonomial graph, denoted G_n. Entries of the adjacency matrix of G_n depend on the well-known Fibonomial coefficients modulo 2. We investigate the connectivity properties, eigenvalues, and energy of G_n. We lastly obtain the sum of the Laplacian eigenvalues of G_n.

1. Introduction

Christopher and Kennedy [5] introduced binomial graphs whose entries are dependent on the well-known binomial coefficients. For $0 \leq n \in \mathbb{Z}$, the binomial graph with 2^n vertices is denoted by B_n. The vertex set V_n and the edge set E_n of the binomial graph B_n are defined by $V_n = \left\{v_j : j = 0, 1, \ldots, 2^n - 1\right\}$ and $E_n = \left\{\left(v_i, v_j\right) : \binom{i+j}{j} \equiv 1 (\text{mod } 2)\right\}$. Christopher and Kennedy also investigated the spectrum and closed walks of B_n.

The well-known Fibonacci sequence $\{F_n\}_{n=0}^{\infty}$ is defined by the recurrence relation $F_{n+2} = F_{n+1} + F_n$ with initial conditions $F_0 = 0$ and $F_1 = 1$. From the Fibonacci recurrence relation, we can see that F_{3k} is always even and all other Fibonacci numbers are odd for $0 \leq k \in \mathbb{Z}$. Fibonacci numbers have many applications in the field of graph theory (see [19,21,13,24]). Detailed information on the Fibonacci numbers and their properties can be found in [14].

Similar to the binomial coefficients, Fibonomial coefficients have been widely used (see [10,22,11,17,18]). Fibonomial coefficients are defined for $n \geq k \geq 1$ as

$$\begin{bmatrix} n \\ k \end{bmatrix}_F = \frac{F_n F_{n-1} \cdots F_{n-k+1}}{F_1 F_2 \cdots F_k} \tag{1}$$

with $\begin{bmatrix} n \\ 0 \end{bmatrix}_F = 1$ and $\begin{bmatrix} n \\ k \end{bmatrix}_F = 0$ for $n < k$. As a consequence of the formula

$$F_m = F_{k+1} F_{m-k} + F_k F_{m-k-1},$$

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which can be shown by an induction argument and the recursion formula

$$\begin{bmatrix} m \\ k \end{bmatrix}_F = F_{k+1} \begin{bmatrix} m-1 \\ k \end{bmatrix}_F + F_{m-k-1} \begin{bmatrix} m-1 \\ k-1 \end{bmatrix}_F .$$

Therefore, (1) will always take integer values [17].

Detailed information about the Fibonomial coefficients is presented in several recent papers [3,12,23,20,16,15]. For the sake of simplicity, we will denote the Fibonomial coefficients by

$$\begin{bmatrix} n \\ k \end{bmatrix}_F = F_{[n,\,k]}.$$

Let $A = \left[a_{ij}\right]$ be a $m \times n$ matrix and $B = \left[b_{ij}\right]$ be a $s \times t$ matrix. The Kronecker product of A and B is denoted by $A \otimes B$.

Let G_n be a graph with the vertex set $V_n = \{v_1,\ v_2, \ldots,\ v_n\}$. The adjacency matrix $A_n = \left[a_{ij}\right]$ of the graph G_n is defined as

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is adjacent to } v_j, \\ 0 & \text{if } v_i \text{ is not adjacent to } v_j, \end{cases}$$

and the eigenvalues of A_n are also the eigenvalues of G_n.

The Laplacian matrix L_n of G_n is defined as $L_n = D_n - A_n$ where D_n is a degree matrix.

Let $\lambda_1,\ \lambda_2, \ldots,\ \lambda_n$ denote the eigenvalues of A_n. In [8], Gutman defines the energy of G_n with n vertices as

$$\varepsilon\left(G_n\right) = \begin{cases} \displaystyle\sum_{i=1}^{n} \left|\lambda_i\right| & \text{if } G_n \text{ is a simple graph,} \\ \displaystyle\sum_{i=1}^{n} \left|\lambda_i - \frac{s}{n}\right| & \text{otherwise,} \end{cases} \tag{2}$$

where $S = tr\left(A_n\right) = \sum_{i=1}^{n} \lambda_i$.

For further references on the energy of a graph and Laplacian energy, see [7,9].

If $\mu_1,\ \mu_2, \ldots,\ \mu_n$ denote the eigenvalues of L_n, then the sum of the Laplacian eigenvalues of G_n is defined as

$$S\left(G_n\right) = \sum_{i=1}^{n} \mu_i .$$

Notations and concepts from graph theory that are not defined herein can be found in [4,6,1].

2. Fibonomial graphs

For $0 \le n \in \mathbb{Z}$, we define a Fibonomial graph $G_n = (V_n,\ E_n)$ with $3 \cdot 2^n$ vertices, G_n, with the vertex set $V_n = \{v_t : t = 0,\ 1,\ 2, \ldots,\ 3 \cdot 2^n - 1\}$ and the edge set $E_n = \left\{\left(v_i,\ v_j\right) : F_{[i+j,\,j]} \equiv 1 (\text{mod } 2)\right\}$. The adjacency matrix of G_n is defined as $A_n = \left[a_{i,j}\right]_{i,j=0}^{3\cdot 2^n - 1}$, where

$$a_{i,j} \equiv F_{[i+j,\,j]} (\text{mod } 2). \tag{3}$$

Below, we give three Fibonomial graphs and their partitioned adjacency matrices in Figs. 1 and 2, respectively.

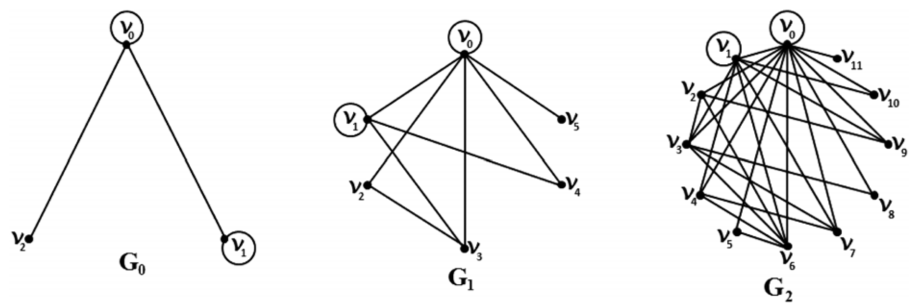

Fig. 1. Fibonomial graphs G_0, G_1 and G_2.