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Fault-free Hamilton cycles in burnt pancake graphs with conditional edge faults

Xiaolan Hu [a], Huiqing Liu [b], Xiang-Feng Pan [c,*]

[a] Department of Mathematics, Nanjing University, Nanjing 210093, China
[b] Faculty of Mathematics and Computer Science, Hubei University, Wuhan 430062, China
[c] School of Mathematical Sciences, Anhui University, Hefei 230601, China

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ABSTRACT

In this paper, we consider the edge fault tolerance of n-dimensional burnt pancake graph BP_n such that each vertex is incident with at least two fault free edges. Based on this requirement, we show that BP_n contains a fault free Hamilton cycle even it has up to $2n-5$ link faults, where $n \geq 3$.

1. Introduction

The performance of an interconnection network highly relies on its interconnection topology. In recent decades, a lot of network topologies are based on the Cayley graphs and have been the focus of many intensive research efforts [1,2,8,9]. A burnt pancake graph [10,11] is such a new topology. Recently, many interesting properties of the burnt pancake graphs are studied (see [5,6,15–19]). Since faults may occur to networks, it is significant to consider faulty networks. In the event of a random edge failure, it is very unlikely that all of the edges incident with a single vertex fail simultaneously. This reason has motivated the research on the Hamiltonian connectivity of *conditional faulty* networks in which each vertex is incident with at least two fault free edges. Based on this requirement, the k-ary n-cube (star graph, alternating group graph, locally twisted cube, crossed cube and pancake graph, resp.) with up to $4n-5$ ($2n-7$, $4n-13$, $2n-5$, $2n-5$ and $2n-7$, resp.) edge faults contains a fault free Hamilton cycle [3] ([7,20,12,14,21], resp.), where $n \geq 3$ is the dimension of the mentioned network. In this paper, we show that the conditional faulty n-dimensional burnt pancake graph contains a fault free Hamilton cycle even it has up to $2n-5$ edge faults, where $n \geq 3$.

2. Preliminaries

Network topology is always represented by a graph where vertices represent processors and edges represent links between processors. All graphs considered here are finite and simple. Undefined terminology and notation may refer to [4]. Let $G = (V, E)$ be an undirected graph and $v \in V(G)$. $N_G(v)$ denotes the neighbor of v in G and $d_G(v) = |N_G(v)|$. Let $\delta(G) =$

* Corresponding author. Tel.: +86 13855125189.
E-mail addresses: xlh_2009@163.com (X. Hu), hql_2008@163.com (H. Liu), xfpan@ustc.edu (X.-F. Pan).

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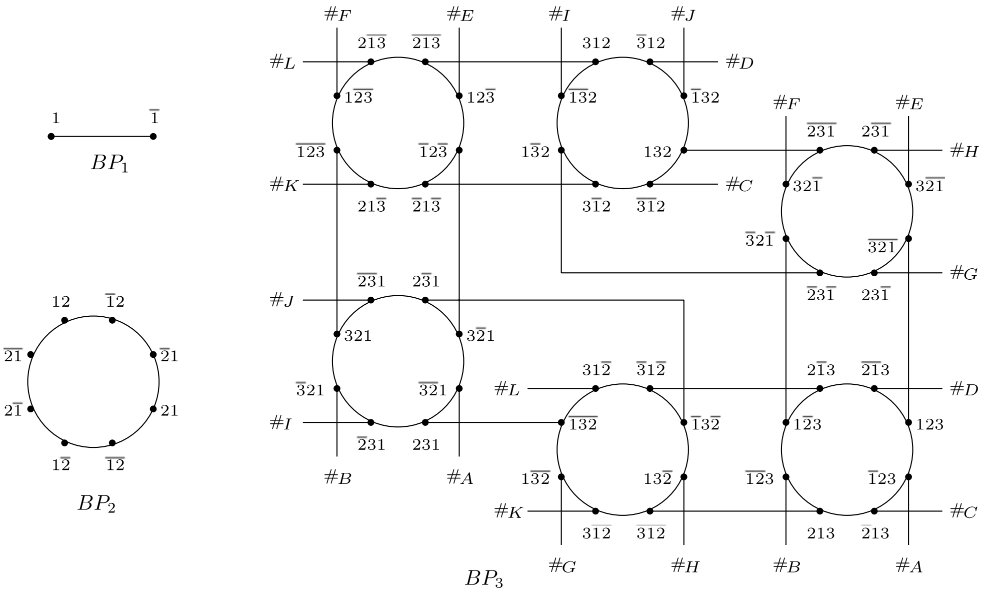

Fig. 1. BP_1, BP_2 and BP_3.

$\min\{d_G(v) : v \in V(G)\}$. For a subgraph H of G, let $N_H(v) = N_G(v) \cap V(H)$ and $d_H(v) = |N_H(v)|$. For $S, T \subseteq V(G)$, $E_G(S, T) = \{uv \in E(G) : u \in S, v \in T\}$, $E_G(S) = E_G(S, V \setminus S)$ and $E_G(u) = E_G(\{u\})$. A path from u to v is called a (u, v)-path. The *distance* between two vertices u and v, denoted by $d_G(u, v)$, is the length of the shortest (u, v)-path. A path or cycle is Hamiltonian if it contains all vertices of the graph.

Let C be a cycle in G with a given orientation. For $v \in V(C)$, we use v^+, v^- to denote the successor and predecessor of v on C, respectively. If $u, v \in V(C)$, we denote by uCv the subpath $uu^+ \cdots v^-v$ of C. The same subpath, in reverse order, is denoted by $v\overline{C}u$. We also use analogous notations for a path P in G.

Let p be an integer, and write the integer $-p$ as $\overline{p}$, to save place. Let $\langle n\rangle = \{1, 2, \ldots, n\}$ and $[n] = \{1, 2, \ldots, n, \overline{1}, \overline{2}, \ldots, \overline{n}\}$. *A signed permutation* of $\langle n\rangle$ is an n-permutation $u_1u_2 \cdots u_n$ of $[n]$ such that the set of absolute value of each element $\{|u_1|, |u_2|, \ldots, |u_n|\} = \langle n\rangle$. For a signed permutation $u = u_1u_2 \cdots u_n$ of $\langle n\rangle$, the *i-th prefix reversal* of u, denoted by $u^{(i)}$, is $\overline{u_i}\,\overline{u_{i-1}} \cdots \overline{u_1}u_{i+1} \cdots u_n$. For example, let $u = 1\overline{2}4\overline{3}5$, then u is a signed permutation of $\langle 5\rangle$, $u^{(2)} = 2\overline{1}4\overline{3}5$ and $u^{(5)} = \overline{5}3\overline{4}2\overline{1}$.

The *n-dimensional burnt pancake graph BP_n* is an n-regular graph with $n!2^n$ vertices, each of which has a unique label from the signed permutation of $\langle n\rangle$. Two vertices u and v are adjacent in BP_n if and only if $u^{(i)} = v$ for some unique i $(1 \le i \le n)$. Such an edge uv is called an i-dimensional edge and v is called the i-neighbor of u. It is seen that every vertex has a unique i-neighbor for $1 \le i \le n$. For convenience, we use $\langle u\rangle_i$ to denote the ith leftmost digit, i.e., $\langle u\rangle_i = u_i$ if $u = u_1u_2 \cdots u_n$, where $1 \le i \le n$. For $r \in [n]$, the subgraph of BP_n induced by the set of vertices u with $\langle u\rangle_n = r$ forms a BP_{n-1}, denoted by $BP_n^{(r)}$. Thus, BP_n contains $2nBP_{n-1}$'s, i.e., $BP_n^{(1)}, BP_n^{(2)}, \ldots, BP_n^{(n)}, BP_n^{(\overline{1})}, BP_n^{(\overline{2})}, \ldots, BP_n^{(\overline{n})}$. Fig. 1 illustrates $BP_3^{(1)}, BP_3^{(2)}, BP_3^{(3)}, BP_3^{(\overline{1})}, BP_3^{(\overline{2})}, BP_3^{(\overline{3})}$ that are embedded in BP_3.

In the following, we always assume $n \ge 3$ and divide BP_n into $BP_n^{(r)}$, $r \in [n]$ along dimension n. For any $p, q \in [n]$, $I \subseteq [n]$, $F \subset E(BP_n)$, we write:

- $E^{(p)}$ for the set of all p-dimensional edges in BP_n;
- $F^{(p)}$ for the edge set $F \cap E(BP_n^{(p)})$;
- $F(n)$ for the edge set $E^{(n)} \cap F$;
- F^I for the edge set $\bigcup_{i\in I} F^{(i)}$;
- $E_{p,q}$ for the edge set $E_{BP_n}(V(BP_n^{(p)}), V(BP_n^{(q)}))$;
- $E_{p,q}^{(r)}$ for the edge set $\{uv \in E(BP_n^{(r)}) : u^{(n)} \in V(BP_n^{(p)}), v^{(n)} \in V(BP_n^{(q)})\}$, where $r \ne p, q$;
- $\widetilde{E}_{p,q}$ for the edge set $E_{BP_n - F}(V(BP_n^{(p)}), V(BP_n^{(q)}))$;
- BP_n^I for the subgraph of BP_n induced by $\bigcup_{i\in I} V(BP_n^{(i)})$;
- $E_p^{Q(q)}$ for the edge set $\{uv \in E(Q(q)) : u^{(n)} \in V(BP_n^{(p)})\}$, where $Q(q)$ is a Hamilton (x, y)-path of $BP_n^{(q)}$ with $x^{(n)}, y^{(n)} \notin V(BP_n^{(p)})$ or $Q(q)$ is a Hamilton cycle of $BP_n^{(q)}$. Note that $E_p^{Q(\overline{p})} = \emptyset$.

Lemma 2.1 ([13]). *Let $u \in V(BP_n^{(p)})$ and $v \in V(BP_n^{(q)})$ with $u \ne v$ and $p, q \in [n]$.*

(i) *If $p = q$ and $d_{BP_n}(u, v) \le 2$, then $\langle u^{(n)}\rangle_n \ne \langle v^{(n)}\rangle_n$;*
(ii) *If $p \ne q$ and $d_{BP_n}(u, v) \le 3$, then $\langle u^{(n)}\rangle_n \ne \langle v^{(n)}\rangle_n$.*