Discrete Applied Mathematics 164 (2014) 47–60

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Fractional routing using pairs of failure-disjoint paths

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ARTICLE INFO

Article history:
Received 21 October 2010
Received in revised form 2 December 2011
Accepted 14 December 2011
Available online 10 January 2012

Keywords:
Shortest paths
Disjoint paths
Compact formulations
Column generation
Capacitated network design

ABSTRACT

Given a set of commodities and a network where some arcs can fail while others are reliable, we consider a routing problem with respect to a survivability requirement that each commodity can be split among pairs of failure-disjoint paths. Two paths p and p' form a pair of failure-disjoint paths if they share only reliable arcs. The same flow is sent over p and p', but the flow sent on a common reliable arc is not doubled.

We present a compact linear formulation of the problem. Also three non-compact formulations solvable by column generation are introduced. In the first formulation, the generated columns correspond to pairs of failure-disjoint paths, while in the second formulation the generated columns correspond to simple paths. The third formulation is solved by generating pairs of arc-disjoint paths. All formulations are compared numerically. On top of that we study some generalizations and some special cases of the problem of computing a shortest pair of failure-disjoint paths. One of these generalizations is equivalent to a single-commodity capacitated network design problem.

1. Introduction

Consider a network given by a directed graph $\mathcal{N} = (\mathcal{V}, \mathcal{A})$ and a capacity c_a for each arc $a \in \mathcal{A}$. Let $\mathcal{K}$ be the set of commodities (demands). Each commodity $k \in \mathcal{K}$ has a value d_k and should be carried from a source s_k to a sink (destination) t_k. We use either $\mathcal{P}_k$ or $\mathcal{P}_{s_k t_k}$ to denote the set of paths joining s_k to t_k, and $\mathcal{P}$ to denote the set of all paths. The classical fractional routing problem consists in determining for each commodity k and each path $p \in \mathcal{P}_k$ the fraction f_p of d_k that will be carried on this path such that all capacity requirements are satisfied:

$$\sum_{p \in \mathcal{P}, p \ni a} f_p \leq c_a, \quad \forall a \in \mathcal{A},$$

and all commodities are routed:

$$\sum_{p \in \mathcal{P}_k} f_p \geq d_k, \quad \forall k \in \mathcal{K}.$$

Different objective functions can be considered for the above construction. When the objective function is linear, the fractional routing problem is simply a linear program.

Fractional routing is well adapted to communication networks when all network links are perfectly reliable. However, in real-world networks some links can fail which makes the fractional routing infeasible. Then some mechanisms have to

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0166-218X/$ – see front matter
doi:10.1016/j.dam.2011.12.019

be introduced to deal with network failures [16,17]. $1+1$ protection is one of these mechanisms with pairs of arc-disjoint paths used instead of paths. More precisely, for each pair of arc-disjoint paths $\{p, p'\}$ connecting s_k and t_k, a certain value of traffic $f_{pp'}$ is simultaneously sent on p and p'. If a link belonging to p fails, then we are sure that the sink can receive traffic through path p' if we assume that link failures cannot be simultaneous. Let us write $p \cap p' = \emptyset$ to say that p and p' are arc-disjoint. Then the traffic condition becomes

$$\sum_{p,p' \in \mathcal{P}_k, p \cap p' = \emptyset} f_{pp'} \geq d_k, \quad \forall k \in \mathcal{K}$$

and the capacity constraint is modified in a similar way:

$$\sum_{k \in \mathcal{K}} \sum_{p,p' \in \mathcal{P}_k, p \cap p' = \emptyset, p \cup p' \ni a} f_{pp'} \leq c_a, \quad \forall a \in \mathcal{A}.$$

As before, if the objective function is linear, we get a linear program easily solvable by column generation (columns correspond to pairs of arc-disjoint paths).

For the case when one commodity is considered and the goal is to maximize the flow to be carried from the source to the sink, a combinatorial algorithm is proposed in [13,14]. Some algorithmic improvements are presented in [1] in addition to a compact linear formulation for the maximum flow problem. In fact, the results of [1,13,14] are also valid if we consider l arc-disjoint paths ($l \geq 2$).

Real-world communication networks are generally made up of different layers, for example, a SONET/SDH layer over an optical network layer. A link of the SONET/SDH layer can be considered as a demand that should be routed through a path in the optical layer. If this path is protected against failures, then the link is reliable. Otherwise the link can fail and hence requires some protection against failures at the SONET/SDH layer [16,17]. This leads to two kinds of arcs: perfectly reliable arcs that do not fail, and unreliable arcs that can fail. Let $\mathcal{R}$ be the set of reliable arcs and $\mathcal{F} = \mathcal{A} \setminus \mathcal{R}$ be the set of unreliable arcs. Then, it becomes clear that instead of using arc-disjoint paths, we should rather consider pairs of failure-disjoint paths (i.e., pairs of paths that can share only reliable arcs). Let us use notation $p \cap p' \cap \mathcal{F} = \emptyset$ when p and p' are failure-disjoint paths. By q we will denote a pair of failure-disjoint paths $\{p, p'\}$, while the set of all failure-disjoint pairs of paths for demand k will be denoted by $\mathcal{Q}_k$. The demand constraint becomes

$$\sum_{q \in \mathcal{Q}_k} f_q \geq d_k, \quad \forall k \in \mathcal{K}$$

while capacity constraint becomes

$$\sum_{k \in \mathcal{K}} \sum_{q \in \mathcal{Q}_k, q \ni a} f_q \leq c_a, \quad \forall a \in \mathcal{A}.$$

When the objective function is linear, we get a linear program with potentially an exponential number of variables. If we try to solve it by column generation, then we have to solve a sub-problem to generate shortest pairs of failure-disjoint paths. This problem can be solved using a combinatorial polynomial-time algorithm of [19] that will be summarized in Section 3.

We will also propose a compact formulation for the problem. This compact formulation can be decomposed in two ways leading to two other non-compact formulations where columns can be generated by shortest path computations for the first formulation, and by computing pairs of arc-disjoint paths for the second formulation.

The paper is organized as follows. In Section 2, the problem is formally presented. For the sake of simplicity, we concentrate on the single-commodity case only (while all formulations and algorithms apply to the multicommodity case). Several formulations and polynomial-time algorithms solving the problem are studied in Section 3. Numerical results are presented in Section 4. Some generalizations and special cases are proposed in Section 5. Conclusions follow in Section 6.

2. Problem

Consider a problem of computing the least congested routing of a given demand on pairs of failure disjoint paths in a capacitated network. For the sake of simplicity we assume that there is only one demand, thus we skip k indices for appropriate constants and variables. By Z we denote the congestion, i.e., the maximum fraction of capacity occupied on any link. The problem is formally stated in (1).

$$\min Z$$

$$\sum_{q \in \mathcal{Q}} f_q \geq d \quad [w] \tag{1a}$$

$$\sum_{q \in \mathcal{Q}, q \ni a} f_q \leq Z \cdot c_a, \quad \forall a \in \mathcal{A} \quad [\pi_a] \tag{1b}$$

$$f_q \geq 0, \quad \forall q \in \mathcal{Q}. \tag{1c}$$