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Robust location transportation problems under uncertain demands

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ABSTRACT

In robust optimization, the *multi-stage* context (or dynamic decision-making) assumes that the information is revealed in stages. So, part of the decisions must be taken before knowing the real values of the uncertain parameters, and another part, called *recourse decisions*, is taken when the information is known. In this paper, we are interested in a robust version of the location transportation problem with an uncertain demand using a 2-stage formulation. The obtained robust formulation is a convex (not linear) program, and we apply a cutting plane algorithm to exactly solve the problem. At each iteration, we have to solve an NP-hard recourse problem in an exact way, which is time-consuming. Here, we go further in the analysis of the recourse problem of the location transportation problem. In particular, we propose a mixed integer program formulation to solve the quadratic recourse problem and we define a tight bound for this reformulation. We present an extensive computation analysis of the 2-stage robust location transportation problem. The proposed tight bound allows us to solve large size instances.

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1. Introduction

Robust optimization is a recent methodology for handling problems affected by uncertain data, and where no probability distribution is available on the ambiguous parameters.

The goal of the robust optimization framework is to obtain a "robust" solution, which protects the decision maker against adverse realizations of the uncertainty. A specific definition of robustness depends on the modeling of the uncertainty, the location of the uncertainty within the problem (in the objective function and/or in the constraints), and the decision-making context (in particular, the way information is revealed over time). Within the problem under consideration in this paper, the uncertainty concerns the constraints, or in other words the feasibility of a solution.

According to the decision-making context, the robust approaches can be divided into two categories. The first category is suited to the *single-stage* context, where the decision-maker has to select a solution before knowing the real value of each uncertain parameter. When uncertainty affects the *feasibility* of a solution (because constraint coefficients are uncertain), robust optimization seeks to obtain a solution that will be feasible for any realization taken by the unknown coefficients (Soyster [15]); however, complete protection from adverse realizations often comes at the expense of a severe deterioration in the objective. This extreme approach can be justified in some engineering applications of robustness, such as robust control theory, but is less advisable in operations research, where adverse events such as low customer demand do not produce the high-profile repercussions that engineering failures – such as a doomed satellite launch or a destroyed unmanned robot – can have. To make the robust methodology appealing to business practitioners, robust optimization thus focuses on obtaining a solution that will be feasible for any realization taken by the unknown coefficients within a smaller, "realistic" set, called the *uncertainty set*. The specific choice of the set plays an important role in ensuring computational tractability of the robust problem and limiting deterioration of the objective at optimality, and must be thought through

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carefully by the decision maker (see for example [2]). In the approach proposed by Bertsimas and Sim [6,5] the assumption is that only a subset of parameters should simultaneously reach their worst value. Thus, the uncertainty set is defined by the cardinality Γ of this subset, given by the decision maker, and called the budget of uncertainty.

The second category is suited to the *multi-stage* context (or dynamic decision-making) where the information is revealed in subsequent stages. The multi-stage approach in robust optimization was first introduced by Ben-Tal et al. [3], and initially focused on two-stage decision making on linear programs under an uncertain feasible set. The idea is to consider two sets of variables, such that the first set must be determined before the disclosure of the uncertainty, and that the second one can be computed after the uncertainty has been revealed. The second stage problem is known as the recourse problem. The 2-stage robust methodology was inspired from stochastic optimization. In the latter case, the uncertainty is described by probability laws, and one has to decide the "here-and-now" variables using an expected value of all possible recourse decisions. The "wait-and-see" variables are decided after the uncertainty has been disclosed. This argument is extended to robust optimization. Indeed, recalling that there are no probability estimations available on the uncertain parameters, one has to decide on the first stage variables making possible a recourse decision for any possible realization of the uncertain parameters in the uncertainty set.

In a recent paper, Bertsimas et al. [4] proposed a complete overview on the theory and the applications of robust optimization. In the specific application considered in this paper, which is the problem of choosing the location of warehouses and the transportation scheme from warehouses to customers, only the right hand side coefficients (representing the demands) are uncertain. This particular context has been studied by Thiele et al. [16]. The authors describe a two-stage robust approach to address general linear programs affected by an uncertain right hand side. The robust formulation they obtained is not a linear but a convex problem, and they propose a cutting plane algorithm to exactly solve the problem. Indeed, at each iteration, they have to solve an NP-hard recourse problem in an exact way, which is time-consuming.

In this paper, our objectives are to propose a robust version of the location transportation problem with uncertain demands, using a 2-stage formulation, and to apply the algorithm proposed by Thiele et al. [16]. Here, we go further in the analysis of the recourse problem of the location transportation problem. In particular, we define a tight bound for a mixed-integer reformulation.

The paper is organized as follows: in Section 2, the location transportation problem is introduced along with its corresponding 2-stage robust formulation. A mixed integer program is then proposed in Section 3 to solve the quadratic recourse problem with a tight bound. Finally, in Section 4, the results of numerical experiments are discussed.

2. Robust location transportation problem

We consider the following location transportation problem: a commodity has to be transported from each of m potential sources to each of n destinations. The sources capacities are $C_i,\ i = 1, \ldots, m$ and the demands at the destinations are $\beta_j,\ j = 1, \ldots, n$. To guarantee feasibility, we assume that the total sum of the capacities at the sources is greater than or equal to the sum of the demands at the destinations. The fixed and variable costs of supplying from source $i = 1, \ldots, m$ are f_i and d_i, respectively. The cost of transporting one unit of the commodity from source i to destination j is μ_{ij}. The goal is to determine which sources to open (r_i), the supply level y_i and the amounts t_{ij} to be transported such that the total cost is minimized. The mathematical formulation of the nominal location transportation problem is the following linear program (T):

$$(T)\begin{cases} \min & \displaystyle\sum_{i=1}^{m} d_i y_i + \sum_{i=1}^{m} f_i r_i + \sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij} t_{ij} & \\ \text{s.t.} & \displaystyle\sum_{j=1}^{n} t_{ij} \le y_i & i = 1, \ldots, m \\ & \displaystyle\sum_{i=1}^{m} t_{ij} \ge \beta_j & j = 1, \ldots, n \\ & y_i \le C_i r_i & i = 1, \ldots, m \\ & r_i \in \{0, 1\}, \qquad y_i, \qquad t_{ij} \ge 0 & i = 1, \ldots, m,\ j = 1, \ldots, n. \end{cases}$$

Furthermore, it should be noted that the decision maker has to decide in two steps: first, the warehouses have to be located and filled, and after, once the demands are known, the routing of commodities is decided. According to this context, the transportation part of the problem has a significant importance when deciding the location part.

In practice, the customers' demands are often estimated at the stage of construction of the warehouses. To be realistic, it is common to assume some uncertainty in these demands. We define the uncertainty set as being interval numbers for each one of them. Formally, the jth customer demand β_j belongs to $[\overline{\beta}_j - \hat{\beta}_j, \overline{\beta}_j + \hat{\beta}_j]$, where $\overline{\beta}_j \ge 0$ represents the nominal value of β_j and $\hat{\beta}_j \ge 0$ its maximum deviation. Clearly, each demand β_j can take on any value from the corresponding interval, regardless of the values taken by other coefficients. We denote as (T^β) the location transportation problem for a given $\beta \in [\overline{\beta} - \hat{\beta}, \overline{\beta} + \hat{\beta}]$, with a nonempty feasible set. Finally, we denote as $\text{opt}(T^\beta)$ the optimal value (bounded value) of (T^β) for a given β.

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