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On Minimum Reload Cost Cycle Cover

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ABSTRACT

We consider the problem of spanning the nodes of a given colored graph $G = (N, A)$ by a set of node-disjoint cycles at minimum reload cost, where a non-negative reload cost is paid whenever passing through a node where the two consecutive arcs have different colors. We call this problem Minimum Reload Cost Cycle Cover (MinRC3 for short). We prove that it is strongly NP-hard and not approximable within $\frac{1}{\epsilon}$ for any $\epsilon > 0$ even when the number of colors is 2, the reload costs are symmetric and satisfy the triangle inequality. Some IP models for MinRC3 are then presented, one well suited for a Column Generation approach. The corresponding pricing subproblem is also proved strongly NP-hard. Primal bounds for MinRC3 are obtained via local search based heuristics exploiting 2-opt and 3-opt neighborhoods. Computational results are presented comparing lower and upper bounds obtained by the above mentioned approaches.

1. Introduction

We consider an optimization problem defined on a graph $G = (N, A)$, either directed or undirected, where all arcs have received a color from a set L of cardinality h, and where it is known that all nodes can be spanned by a set of node-disjoint cycles. A *reload cost* is paid along a path or cycle in G whenever passing through a node where the two consecutive arcs have different colors. All possible values of reload costs are given by a *reload cost function* $r : L \times L \to \mathbb{Z}^+$, where $r(l, l')$ denotes the reload cost that arises at a node when passing from color l of one incident arc to color l' of the other. We define the reload cost of a cycle in G as the sum of the reload costs that arise at its nodes. We are interested in spanning all nodes of G by a set S of node-disjoint cycles so as to minimize the sum of the reload costs of the cycles in S. We call this problem Minimum Reload Cost Cycle Cover (MinRC3).[1]

The concept of reload costs was introduced in the seminal paper [11] where various applications are mentioned. This very natural concept, which models a kind of cost incurred during a transportation/transmission activity, has been, amazingly, considered only recently, so that the list of relevant references is comparatively small [1,5,6,9].

We will be interested in asymmetric reload costs and in reload costs that satisfy the *triangle inequality*, where, for undirected graphs, we say that the reload costs satisfy the triangle inequality if, for any three edges e, e', e'' incident in a node of the graph, colored, respectively, with colors l, l', l'', we have that $r(l, l') \le r(l, l'') + r(l'', l')$. We will denote the color of an arc $e = (i, j)$ by c_e or $c_{i,j}$.

MinRC3 is solvable in polynomial time when the number h of colors is equal to 1, since in this case we are left with the well known problems of finding a spanning 2-matching of G if the graph is undirected, or a set of directed cycles spanning N if the graph is directed.

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[1] For standard definitions on graph related concepts, see for instance [3].

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In this work we analyze the complexity of MinRC3, we present some integer programming formulations, and we report on preliminary computational results. In Section 2 we prove that, even when $h = 2$, the reload costs are symmetric and satisfy the triangle inequality, MinRC3 is strongly NP-hard and it is not approximable within $\frac{1}{\epsilon}$, for any $\epsilon > 0$. Moreover we show that even in this case, if we additionally require that $r(l, l) \geq 1$ for any $l \in L$, then the problem is not approximable within $O(2^{p(n)})$, for any polynomial $p()$. In Section 3 we present a bilinear formulation, its standard linearized version, and a formulation suitable for a Column Generation approach. In Section 4 we describe a local search approach for obtaining primal bounds. In Section 5 some preliminary computational results are reported. Ongoing work and directions for further research are also discussed.

2. Complexity

In this section we analyze the complexity and the approximability of problem MinRC3 formulated on undirected graphs. The results are presented in Theorem 1, Corollaries 1 and 2. It is straightforward to show that these results also hold for MinRC3 formulated on directed graphs.

The first result of this section derives from a reduction of the well known Vertex Cover problem to the recognition form of MinRC3, which we call Reload Cost Cycle Cover(RC3). We begin by giving the definitions of these problems and by presenting the reduction.

Vertex Cover: An *instance* I of Vertex-Cover consists of an undirected graph $G = (V, E)$ and a positive integer k. The *question* is whether there exists a subset S of V having $|S| \leq k$ and covering all edges in E, i.e. such that for each edge $e \in E$ one of its end vertices belongs to S.

Reload Cost Cycle Cover: An *instance* I' of RC3 consists of an undirected graph $G' = (N, A)$, a finite set L of colors, a function $c : A \to L$ that assigns a color to each edge, a reload cost function $r : L \times L \to \mathbb{Z}^+$, and a positive integer k'. The *question* is whether there exists a set C of node-disjoint cycles spanning the set N of nodes and having a reload cost at most equal to k'.

The reduction that we now propose for building in polynomial time, given an instance I of Vertex Cover, a corresponding instance I' of RC3, is a modification of the reduction described in [7] that reduces Vertex Cover to Hamiltonian Circuit. We do not describe here the entire reduction, since it is practically identical to the very well known one presented in [7], to which we refer and to which we invite the reader to refer also for the notations, but we focus our attention only on the differences in the two reductions that allow us to conclude that Lemma 1 holds. The difference among the reductions lies in the so called "cover-testing component" for an edge $e = \{u, v\}$ of G and, of course, in the coloring of the edges of G'.

In Fig. 1, for sake of clarity, we illustrate the cover-testing component used in [7] and in our reduction. In our reduction, for each $e = \{u, v\}$ of G there are in G' five new vertices $(u, e, 0)$, $(v, e, 0)$, uv_1, uv_2, uv_3 connected by new edges as illustrated. Notice that in Fig. 1(b) the colors of the edges are also depicted, with edges drawn with light lines having color 1 and edges drawn with heavy lines having color 2. To conclude the presentation of our reduction we specify that all other edges of G' have color 1 so that $L = \{1, 2\}$, the reload cost function r is such that $r(l_1, l_2) = 0$ if $l_1 = l_2$ and $r(l_1, l_2) = 1$ if $l_1 \neq l_2$, and finally the integer k' is set equal to 0.

Lemma 1. *Let I be an instance of* Vertex Cover *and I' be the corresponding instance of* Reload Cost Cycle Cover. *There exists in $G = (V, E)$ a subset S of V having $|S| \leq k$ and covering all edges in E if and only if there exists in $G' = (N, A)$ a set of node-disjoint cycles spanning the set N of nodes and having a reload cost equal to* 0.

Proof. Suppose there exists in G a set S of k vertices covering all edges in E. If we disregard for a while all vertices uv_i, $i = 1, 2, 3$, for each $\{u, v\} = e \in E$, the same reasoning in [7] allows us to assert that there is a cycle, made with edges of color 1, that goes thorough the k "selector" vertices $a_1, \ldots, a_k$, (so called in [7]), and the remaining vertices of G' (except possibly some $(u, e, 0)$ or $(v, e, 0)$); this cycle, within the cover testing component for edge $e = \{u, v\}$, has one of the three possible configurations illustrated in Fig. 2, where (a)–(c) correspond to the cases in which u belongs to S but v does not, both u and v belong to S, v belongs to S but u does not.

From this observation it is straightforward to see that in the three cases it is possible to span all vertices uv_i, $i = 1, 2, 3$, $\{u, v\} = e \in E$ (and the unspanned $(u, e, 0)$ or $(v, e, 0)$) with a node-disjoint cycle of color 2 (also illustrated in Fig. 2), therefore obtaining a set of node-disjoint cycles spanning the nodes of G' and having a reload cost equal to 0. Suppose on the contrary that G' has a set C of node-disjoint cycles spanning the set N of nodes and having reload cost equal to 0. In this case each cycle in C cannot have edges of both colors. Therefore, for each edge $e = \{u, v\}$, the vertices uv_i, $i = 1, 2, 3$ must be spanned by a cycle of the three types illustrated in Fig. 2 and this implies that the remaining vertices (u, e, i), (v, e, i), $i = 1, \ldots, 6$ must be spanned by edges of color 1, again in one of the tree types illustrated in Fig. 2. At this point it is easy to conclude, as done in [7], that any portion of the cycles with edges of color 1 that begins at one selector vertex and ends at a selector vertex without passing through any other selector vertex, corresponds to those edges from E that are incident to some particular vertex in V. The cycles passing through the k selector vertices identify k such portions that identify the k vertices of G that cover all the edges in E. □

Since in the reduction presented in the proof of Lemma 1 the number of colors of the edges of G' is 2, the reload costs are symmetric, and satisfy the triangle inequality, we may conclude that the following theorem is true.