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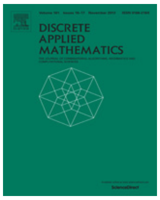

Exact and heuristic solution approaches for the Integrated Job Scheduling and Constrained Network Routing Problem

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ABSTRACT

This paper examines the problem of scheduling a number of jobs on a finite set of machines such that the overall profit of executed jobs is maximized. Each job has a certain demand, which must be sent to the executing machine via constrained paths. A job cannot start before all its demands have arrived at the machine. Furthermore, two resource demand transmissions cannot use the same edge in the same time period. The problem has application in grid computing, where a number of geographically distributed machines work together for solving large problems. The machines are connected through an optical network.

The problem is formulated as an IP problem and is shown to be $\mathcal{NP}$-hard. An exact solution approach based on Dantzig–Wolfe decomposition is proposed. Also, several heuristic methods are developed by combining heuristics for the job scheduling problem and for the constrained network routing problem.

The methods are computationally evaluated on test instances arising from telecommunications with up to 500 jobs and 500 machines. Results show that solving the integrated job scheduling and constrained network routing problem to optimality is very difficult. The exact solution approach performs better than using a standard IP-solver; however, it is still unable to solve several instances. The proposed heuristics generally have good performance. Especially, the First Come First Serve scheduling heuristic combined with a routing strategy, which proposes several good routes for each demand, has good performance with an average solution value gap of 3%. All heuristics have very small running times.

1. Introduction

Heuristic and exact solution methods for the Integrated Job Scheduling and Constrained Network Routing Problem (JSCNR) are presented. The JSCNR consists of scheduling a number of jobs on a finite set of machines such that the overall profit of executed jobs is maximized. Each job has a certain demand, which must be sent to the executing machine via constrained paths. A job cannot start before all its demands have arrived at the machine. Furthermore, two job demand transmissions cannot use the same edge in the same time period. Finally, time windows of both machines and jobs must be obeyed. It is assumed that the set of jobs, the set of machines, and the state of the network are known in advance; hence the problem can be viewed as being *offline*.

The problem has application in distributed production systems, where a set of jobs can be carried out at various plants. If the total job execution exceeds the total number of available machines and if the transportation paths are limited, it is necessary to consider both problems simultaneously. A typical application is the steel industry where the production can be placed at various sites, but the transportation of iron ore and coal by e.g. train, constitutes a substantial logistic problem.

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The problem also has application in grid computing where jobs are to be executed at various grid resources (machines) and where the grid resources are connected through an optical network. A job cannot be executed before its input data has arrived at the executing grid resource, and two data transmissions cannot use the same wavelength on the same fiber at the same time.

An example is The Large Hadron Collider (LHC) Physics Program by The European Organization for Nuclear Research (CERN). It is estimated that the LHC experiments generate 15 petabytes of data annually, thus the project utilizes grid computing not only for distributing the scientific work, but also for distributing data storage. The network connections for the grid computing system must support high bandwidth availability, like e.g. optical networks. For details on the Worldwide LHC Computing Grid, see their homepage [5]. See [3] for a description of optical networks and applications.

The contribution of this paper is to model and solve JSCNR. We show that the problem is $\mathcal{NP}$-hard and propose several heuristic and exact solution methods. The exact solution method is a branch-and-cut-and-price (BCP) algorithm based on applying Dantzig–Wolfe decomposition, see [8], such that the master problem determines where and when jobs are executed and the pricing problem calculates routing schemes. The heuristics are based on combining methods for the Integrated Job Scheduling and Network Routing Problem (JSNR) and for the Constrained Network Routing Problem (CNR).

Two types of test instances are generated: a tandem topology with 10–500 jobs and 10–500 machines and a real-life network topology taken from the Nordic DataGrid Facility with 10–200 jobs and 14 machines. The suggested solution methods are evaluated on the test instances. The BCP algorithm performs better than applying `CPLEX` on an IP formulation; however, it is unable to solve several of the considered test instances within a half hour time frame. The heuristics are capable of solving all instances within minutes. Best general heuristic performance is reached when using the First Come First Serve strategy for JSNR and a routing scheme which suggests two different paths for each demand for CNR. This setting gives an average solution value gap of 3%.

This paper is structured as follows. First JSCNR is defined in Section 2. Related work from the literature is also presented in this section along with notation and a mathematical model. In Section 3 heuristic methods are presented as combinations of methods for JSNR and for CNR. An exact BCP algorithm is presented in Section 4. The suggested solution methods are computationally evaluated in Section 5 and final remarks are given in Section 6.

2. Problem definition

This section defines the Integrated Job Scheduling and Network Routing Problem (JSNR) and the Constrained Network Routing Problem (CNR). The problems are combined into the Integrated Job Scheduling and Constrained Network Routing Problem (JSCNR). For each problem an overview of work in the literature is given.

JSNR consists of scheduling a number of jobs on a finite set of machines such that the overall profit of executed jobs is maximized. Each job has a certain demand, which must be sent to the executing machine via capacitated paths. Several job demand transmissions can travel on the same edge at the same time. A job cannot start before all its demand has arrived at the machine, and time windows of both machines and jobs must be obeyed.

JSNR has application in grid computing where jobs are executed on grid resources and where job input files must be sent to the executing grid resource through a packet-switched network before execution can begin.

JSNR is closely related to JSCNR, but differs in the routing of job demands. Below, we argue that the routing part of JSCNR is $\mathcal{NP}$-hard. The routing part of JSNR, however, is polynomially solvable [12], hence the JSNR may be more tractable than JSCNR.

A simple version of JSNR consisting of sharing bandwidths in grid computing context was proved to be $\mathcal{NP}$-hard and greedy heuristics were presented by Marchal et al. [16].

An offline scheduler consisting of two steps was presented by Agarwal et al. [1]: first jobs were scheduled to grid resources such that the total penalty of delayed job executions was minimized, then the overall starting and end times of job schedules were determined.

Elghirani et al. [9] proposed a tabu search algorithm, which assigned jobs to a set of grid resources. The solution neighborhood consisted of moving a scheduled job to another available grid resource and often used moves were penalized to avoid move cycles. When no improvement was reached in a certain time interval, the tabu list was cleared, a new random solution was found, and the tabu procedure started all over.

JSNR was shown to be $\mathcal{NP}$-hard and solved to optimality by Gamst and Pisinger [12]. The solution method was based on a Dantzig–Wolfe decomposition, where the pricing problem was the polynomial linear multi-commodity flow problem assigning a single job to a single machine. The master problem found an overall feasible solution, and the branching strategy added cuts to strengthen the formulation. Results showed that their algorithm outperformed both simpler exact algorithms and `CPLEX`. The algorithm was capable of solving instances with up to 1000 jobs and 1000 machines within minutes.

The telecommunication application of JSNR was solved heuristically by Gamst [11] using a number of greedy heuristics, a swap-based metaheuristic and the adaptive large neighborhood metaheuristic. Results showed that though the metaheuristics found better solution values than the greedy methods, they also had relatively large running times.

CNR consists of sending demand through a network such that two routes never use the same edge simultaneously. Given is a network consisting of nodes and capacitated edges. The network takes time into account, i.e., an edge can be visited at different time slots. Also given is a set of routing requests each consisting of a source, a destination, a routing time window,