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Natural and extended formulations for the Time-Dependent Traveling Salesman Problem

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ABSTRACT

In this paper, we present a new formulation for the Time-Dependent Traveling Salesman Problem (TDTSP). We start by reviewing well known natural formulations with some emphasis on the formulation by Picard and Queyranne (1978) [22]. The main feature of this formulation is that it uses, as a subproblem, an exact description of the n-circuit problem. Then, we present a new formulation that uses more variables and is based on using, for each node, a stronger subproblem, namely an n-circuit subproblem with the additional constraint that the corresponding node is not repeated in the circuit. Although the new model has more variables and constraints than the original PQ model, the results given from our computational experiments show that the linear programming relaxation of the new model gives, for many of the instances tested, gaps that are close to zero. Thus, the new model is worth investigating for solving TDTSP instances. We have also provided a complete characterization of the feasible set of the corresponding linear programming relaxation in the space of the variables of the PQ model. This characterization permits us to suggest alternative methods of using the proposed formulations.

1. Introduction

Consider a graph $G = (V, A)$, where $V = \{1, 2, \ldots, n\}$ and $A = \{(i, j) : i, j = 1, \ldots, n, i \neq j\}$. The Time-Dependent Traveling Salesman Problem (TDTSP) is to find a minimum cost Hamiltonian circuit, starting and ending on node 1, where arc costs depend on its position in the tour. Thus, to each arc (i, j) in A and each possible position h of the arc in the tour we associate a cost c_{ij}^h. Clearly, an arc $(1, j)$ leaving node 1, which we will also denote by the depot, can be only in position 1 and an arc $(i, 1)$ entering the depot can be only in the last position. Every other arc (i, j), $i, j \neq 1$, can be located in positions $h = 2, \ldots, n-1$.

The TDTSP was motivated by the following one-machine scheduling problem. Consider a set of $n-1$ jobs, corresponding to the nodes in the set $V \setminus \{1\}$, to be performed on a single machine which can handle one job at a time. Transition costs c_{ij}^h occur when job i is to be processed at position h and in addition, is immediately followed by job j. We assume an idle state for the machine corresponding to the initial and final states of the machine and which will be represented by node 1. Then, we have a setup cost for any job j, given by c_{1j}^1, and a cooling cost for any job i given by c_{i1}^n. The problem is to find the cheapest sequence for performing all jobs.

Two special cases of the TDTSP are well known. The most well known case, the Asymmetric Traveling Salesman Problem (ATSP) (see, for instance, [18]), is obtained when for every arc (i, j) we have $c_{ij}^h = c_{ij}$ for every h (that is, the cost of each arc

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does not depend on its position). The other case, the so-called Cumulative Traveling Salesman Problem (CTSP) also known as the traveling deliveryman problem, is obtained by considering $c_{ij}^h = (n-h)c_{ij}$ for every arc (i,j) and every h, where c_{ij} is a given "base" cost. The CTSP models the situation where one wants to minimize the sum of all distances from node 1 to any other node (excluding node 1). This model has applications in machine scheduling and delivery problems where one seeks to minimize the average arrival time at the customer locations. We note that in deliver applications, the cost of an arc (i,j) in position h is defined by $(n-h+1)$ c_{ij} since in this case one wants to compute in addition the total time that the distributor is out of the depot. We note that models presented for one case are quite easily adapted for the other one and the results taken from a small computational experiment performed with the models presented in this paper, indicate that no significant difference arises when either one of the two versions is tried. Thus, we will present our models with the first cost definition mentioned above.

Exact methods for solving the CTSP are described, among others, in [19,6,2] and, more recently, in [3,20,1]. Lucena [19] proposes an algorithm based on a non-linear integer programming formulation by Picard and Queyranne in which lower bounds are obtained from a Lagrangian relaxation and presents computational results for problems up to 30 nodes. A similar approach was followed by Bianco et al. [2] which derive a Lagrangian relaxation scheme from an integer linear programming formulation also proposed by Picard and Queyranne (see next section). They solve instances with up to 35 nodes. Fischetti et al. [6] provide a branch-and-bound algorithm based on a new integer programming formulation. The paper contains results on the cumulative matroid that are used to derive lower bounds. Problems with up to 60 nodes are solved to optimality. Bigras et al. [3] use a branch-and-cut scheme based on a path formulation. This is equivalent to the Picard and Queyranne formulation strengthened with several classes of inequalities that are either taken from the ATSP problem (subtour elimination inequalities and 2-matching inequalities) or taken from the node packing problem. The authors apply this procedure also to the Makespan Problem and to the Total Tardiness Problem. They present results for instances taken from the literature up to 50 nodes. Méndez-Díaz et al. [20] propose a new formulation which uses flow based variables as well as variables from the linear ordering problem. In the scope of a branch-and-cut algorithm they introduce several classes of valid inequalities (which are also shown to be facet defining). They produce computational results for instances with up to 40 nodes. In [1], the authors present an approach that is similar to the one presented by Bigras et al. [3] in the sense that column generation applied to a path model is also used. However, Abeledo et al. [1] use inequalities from the TDTSP. Some of these inequalities are lifted versions of inequalities from the TSP, making their method stronger in theory. They produce results taken from instances with up to 76 nodes. They also provide a polyhedral study of the TDTSP showing that one class of the inequalities used in their method are facet defining.

Several formulations for the TDTSP described in the literature (see Section 2) can be obtained by using the binary variables z_{ij}^h for all $(i,j) \in A$ and $h = 1, \dots, n$, indicating whether or not arc $(i,j) \in A$ is in the hth position of the circuit. A formulation that uses only the z_{ij}^h variables is called a *natural* formulation. Natural formulations will be reviewed on Section 2 with some emphasis on the well known formulation by Picard and Queyranne [22]. The main feature of this formulation is that it uses, as a subproblem, an exact description of the n-circuit problem. An n-circuit is a circuit with n arcs which may repeat nodes and even arcs.

The new models discussed in this paper (see Sections 3 and 5) are built on two features: they (i) use a stronger subproblem, a n-circuit subproblem with the additional constraint that a given node is not repeated in the circuit and (ii) combine these subproblems for all nodes. The new formulation will use extra variables (besides the z_{ij}^h variables) and thus, it will fall in the class of so-called *extended* formulations. Although the model has more variables and constraints than the original PQ model, the results given from our computational experiments show that the linear programming relaxation of the new model gives, for many of the instances tested, gaps that are close to zero. Thus, the new model is worth investigating for solving TDTSP instances, either by using it within available ILP packages or as the subject of determining what inequalities are implied by the linear programming relaxation of the new model and are not redundant in the linear programming relaxation of the Picard and Queyranne model. In fact, this is the topic of Section 4 and we will relate a set of such inequalities with the inequalities described in [1]. We should emphasize that our goal is not to obtain a formulation that provides fast lower bounds. The main aim is to propose a formulation that produces very tight lower bounds permitting us to get more insight on the structure of the problem (e.g., projected inequalities, which subproblems are strong for a given commodity). However, in the conclusions, we will suggest some alternative ways for handling the proposed formulation.

In the following we denote the linear programming relaxation of a given model P by P_L and its linear programming bound by $v(P_L)$. We will use the designation "exact" model for a model whose linear programming relaxation only has integral extreme points. We let $F(P)$ denote the set of feasible solutions of an integer (linear) program P. Given an integer linear programming model P defined on two sets of variables x and z, we denote by $\text{Proj}_x(F(P_L))$ the projection of the polyhedron defined by P_L into the space of the x variables, more precisely, $\text{Proj}_x(F(P_L)) = \{x$: there exist z such that (x,z) is feasible for $P_L\}$.

2. Natural formulations for the TDTSP—the Picard and Queyranne formulation

The well known formulation by Picard and Queyranne [22], denoted by PQ in the sequel, is as follows:

$$\text{minimize} \sum_{(i,j)\in A} \sum_{h=1,\dots,n} c_{ij}^h z_{ij}^h z_{1j}^1 + \sum_{h=2,\dots,n} \sum_{i\in V\setminus\{1\}} z_{ij}^h = 1 \quad \text{for all } j \in V \setminus \{1\} \tag{PQ1}$$