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A continuous strategy to solve a class of mixed optimization problems

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ABSTRACT

This work presents a method to solve a class of discrete optimization problems, including linear, quadratic, convex, and discrete geometric programming problems. The methodology consists of inserting, in the original problem, additional geometric constraints where any viable solution is also discrete. Moreover, a strategy to solve signomial geometric programming problems is developed. Computational results are shown through some examples of facility location, machining economics and economic order quantity problems.

1. Introduction

Geometric Programming is a mathematical programming technique often applied to minimize a class of generalized polynomial functions called signomial functions. The technique was developed in the 60's by Duffin and Peterson focusing on posynomial geometric programming problems, i.e., strictly positive signomial functions. Nevertheless, nowadays the technique is still strongly applied as a method for solving Signomial Geometric Problems (SGP), Quadratic Problems with quadratic constraints, Allocation and Financial Problems, and many others. To these kind of problems, the original approach by Duffin et al. [4] didn't seem to be totally useful and an alternative technique, named condensation, was proposed. To condense posynomial functions means to approximate them using the inequality between arithmetic mean and geometric mean or harmonic mean. This approach can give a solution which is just either a stationary point or a local minimum [11]. Other techniques applied with the same purpose are the global optimization techniques based on cut planes and linearization [8,10,7,6]. An extension of SGP can be seen as a problem where a group of variables has discrete values.

In this work we concentrate our attention on the solution of discrete signomial geometric problems by a continuous approach. We adopt a continuous optimization strategy to make sure that the optimal solution satisfies the initial conditions required. To do so, we add signomials constraints to the original problem, where the viability is possible only for discrete values, transforming the signomial discrete problem into a signomial geometric problem.

This paper is organized as follows: Section 2 presents the general problem of geometric programming, the dual problem, some basic concepts and the discrete posynomial geometric primal problem. Section 3 shows the referred continuous strategy to transform the discrete posynomial geometric problem into a signomial geometric problem. Section 4 presents a global optimization technique to solve the signomial problem defined in Section 3. Finally, Section 5 presents facility location problems and some computational results.

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2. Geometric programming

In this section we introduce posynomial, signomial and discrete signomial geometric programming problems. We also present some signomial constraints for which the possible solutions are strictly discrete.

2.1. Geometric programming problems

A Posynomial Geometric Programming Problem (PGP) is an optimization problem stated as follows:

$$\text{PGP}\quad \begin{array}{ll} \text{Minimize} & g_0(t) \\ \text{Subject to} & g_k(t) \leq 1 \quad k = 1, \dots, p \qquad (1) \\ & t_j > 0 \quad j = 1, \dots, m \qquad (2) \end{array}$$

so that

$$g_k(t) = \sum_{i \in J[k]} c_i \prod_{j=1}^{m} t_j^{a_{ij}} \quad k = 0, 1, \dots, p \tag{3}$$

$$J[k] = \{m_k, m_{k+1}, \dots, n_k\} \quad k = 0, 1, \dots, p \tag{4}$$

$$m_0 = 1, \qquad m_1 = n_0 + 1, \qquad m_2 = n_1 + 1, \dots, m_p = n_{p-1} + 1, \qquad n_p = n.$$

Exponents a_{ij} are arbitrary constants, coefficients c_i are positive, functions g_k are called posynomials, terms $c_i \prod_{j=1}^{m} t_j^{a_{ij}}$ are called existing posynomials terms of the problem and variables t_j are primal variables.

Related to the geometric programming problem we have the Dual Geometric Programming problem (DGP) which is given by:

$$\text{DGP}\quad \begin{array}{ll} \text{Maximize} & u(\delta) \\ \text{Subject to} & \sum_{i \in J[0]} \delta_i = 1 \qquad (5) \\ & \sum_{i=1}^{n} a_{ij}\delta_i = 0 \qquad (6) \\ & \delta_i \geq 0 \qquad (7) \end{array}$$

so that

$$u(x) = \prod_{i=1}^{n} \left(\frac{c_i}{\delta_i}\right)^{\delta_i} \cdot \prod_{k=1}^{p} \lambda_k^{\lambda_k} \tag{8}$$

$$\lambda_k = \sum_{i \in J[k]} \delta_i \tag{9}$$

and $J[k]$ is given by (4).

We must observe that the dual function v is not a concave function, but, on the other hand,

$$f(\delta) = \ln(v(\delta)) = \sum_{i=1}^{n} \{\delta_i \ln(c_i) - \delta_i \ln \delta_i\} + \sum_{k=1}^{p} \left(\sum_{i \in J[k]} \delta_i\right) \ln \left(\sum_{i \in J[k]} \delta_i\right) \tag{10}$$

satisfies that property.

Primal and dual variables of a geometric programming problem are connected by the equation:

$$\delta_i = \lambda_k c_i \prod_{j=1}^{m} t_j^{a_{ij}}, \quad i \in J[k]. \tag{11}$$

Definition 1. A primal geometric programming problem (PGP) is called either consistent, feasible or viable, if there is $t \in \mathbb{R}^m$ satisfying Eqs. (1) and (2). If $g_k(t) < 1$, $k = 1, \dots, p$, then problem (*PGP*) is said to be super-consistent or strictly viable.

A dual geometric programming problem (*DGP*) is said to be either consistent, feasible or viable, if there is $\boldsymbol{\delta} \in \mathbb{R}^n$ satisfying (5)–(7).

Definition 2. A geometric programming problem is said to be canonical if its dual problem has a viable solution strictly positive, i.e., there is $\delta \in \mathbb{R}^n$, $\delta_i > 0$, satisfying (6) and (7)

Definition 3. The difficulty degree d of a geometric programming problem is given by

$$d = n - m - 1 \tag{12}$$

where n is the number of posynomial terms and m is the number of primal variables.