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A 1-local 4/3-competitive algorithm for multicoloring a subclass of hexagonal graphs

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ABSTRACT

We consider a frequency allocation problem, in which we are given a cellular telephone network whose geographical coverage area is divided into cells in which phone calls are serviced by frequencies assigned to them, so that none of the pairs of calls emanating from the same or neighboring cells is assigned the same frequency. The problem is to use the frequencies efficiently, i.e., to minimize the span of frequencies used. The frequency allocation problem can be regarded as a multicoloring problem on a weighted hexagonal graph. In this paper, we present a 1-local 4/3-competitive distributed algorithm for multicoloring a hexagonal graph without certain forbidden configuration (introduced in Šparl and Žerovnik (2010) [7]).

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1. Introduction

The basic problem concerning cellular networks concentrates on assigning sets of frequencies (colors) to transmitters (vertices) in order to avoid unacceptable interference (see [1]). In an ordinary cellular model, the transmitters are centers of hexagonal cells and the corresponding adjacency graph is a subgraph of the infinite triangular lattice. In our model, to each vertex v of the triangular lattice T we assign a non-negative integer $d(v)$, called the *demand* (or *weight*) of the vertex v. A *proper multicoloring* of G is a mapping φ from $V(G)$ to subsets of integers (colors) from $[n] = \{1, 2, \dots, n\}$, such that $|\varphi(v)| = d(v)$ for any vertex $v \in V(G)$ and $\varphi(v) \cap \varphi(u) = \emptyset$ for any pair of adjacent vertices u and v in graph G. The minimal n for which there exists a proper multicoloring of G, denoted by $\chi_m(G)$, is called the *multichromatic number* of G.

In studies concerning cellular networks, the idea of hexagonal graphs arises naturally. Formally, following the notation from [3], the vertices of the triangular lattice T can be described as follows: the position of each vertex is an integer linear combination $x\vec{p} + y\vec{q}$ of two vectors $\vec{p} = (1, 0)$ and $\vec{q} = (\frac{1}{2}, \frac{\sqrt{3}}{2})$. Thus vertices of the triangular lattice may be identified with pairs (x, y) of integers. Two vertices are adjacent when the Euclidean distance between them is 1. Therefore, each vertex (x, y) has six neighbors: $(x-1, y)$, $(x-1, y+1)$, $(x, y+1)$, $(x+1, y)$, $(x+1, y-1)$, and $(x, y-1)$. For simplicity, we refer to the neighbors as *left*, *up-left*, *up-right*, *right*, *down-right*, and *down-left*. We define a *hexagonal graph* $G = (V, E)$ as an induced subgraph of the triangular lattice (see Fig. 1).

A *triangle-free hexagonal graph* is a subgraph of the triangular lattice which does not contain any 3-clique. A *corner* in a triangle-free hexagonal graph is a vertex which has at least two neighbors and none of them are at angle π. A vertex which is not a corner is called a *non-corner* (see Fig. 2).

The multichromatic number is closely related to the *weighted clique number* $\omega(G)$, which is defined as the maximum weight of clique over all cliques in G, where the weight of a clique is the sum of demands on its vertices. Obviously, for any

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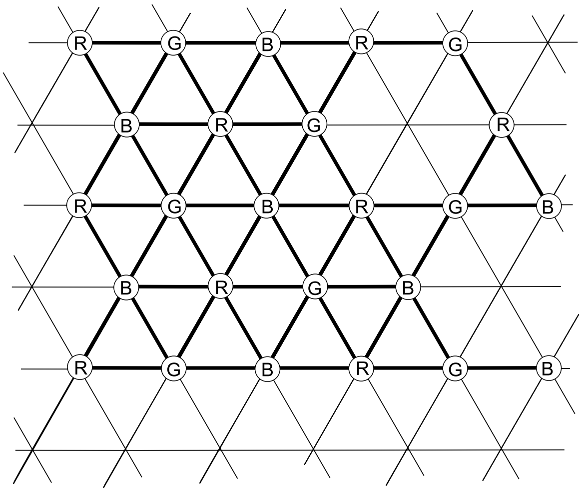

Fig. 1. An example of a hexagonal graph.

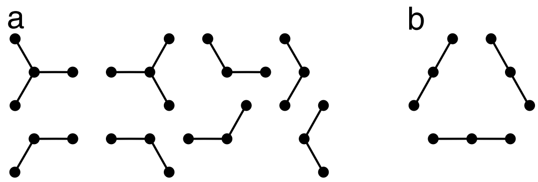

Fig. 2. All possibilities for (a) corners, (b) non-corners.

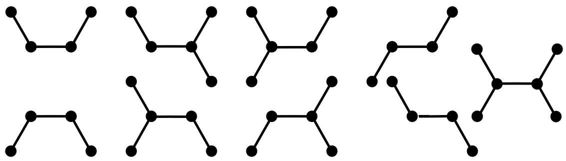

Fig. 3. All possibilities for two horizontal adjacent corners (special heavy double-corner).

graph, $\chi_m(G) \geq \omega(G)$, while for hexagonal graphs we know that $\chi_m(G) \leq \left\lceil \frac{4\omega(G)}{3} \right\rceil + O(1)$ (see [3,5,6]). Since all proofs of the upper bound are constructive, they imply the existence of a 4/3-*competitive algorithm*, i.e. algorithms which can online serve calls with the approximation ratio equal to 4/3 with respect to the weighted clique number (see [2,4]). It should also be mentioned that McDiarmid and Reed proved in [3] that it is an NP-complete problem decide whether $\chi_m(G) = \omega(G)$.

A framework for studying distributed online assignment in cellular networks was developed in [2]. In distributed graph algorithms, a special role is played by the "locality" property. An algorithm is *k-local* if the computation at any vertex v uses only the information about the demands of vertices at distance at most k from v. For hexagonal graphs, the best-known 1-local algorithm for multicoloring is 33/24-competitive, and it has been presented in [8].

In this paper, we develop a new algorithm with ratio 4/3, which works in a 1-local model for hexagonal graphs without a *special heavy double-corner*, defined as follows.

We call a vertex *heavy* if its weight is larger than the average weight of all cliques which contain this vertex. Let G' be a graph induced on G by the heavy vertices in G. A *special heavy double-corner* is a subgraph in G' such that it consist of two adjacent corners in G' and its coordinates (x_1, y_1), (x_2, y_2) satisfy $x_1 \mod 2 \neq y_1 \mod 2$ and $x_2 \mod 2 \neq y_2 \mod 2$ (see Fig. 3).

Triangle-free hexagonal graphs without two adjacent corners (heavy double-corners) were introduced in [7] and [9]. Here, we consider a wider subclass of the hexagonal graphs family than in [7], since we exclude two adjacent corners in one specific position only.

Our algorithm can be extended into a 2-local algorithm for multicoloring an arbitrary hexagonal graph with the ratio 4/3—the same as the best-known algorithm given by Šparl and Žerovnik (see [6]).

We will prove the following theorem.

Theorem 1.1. *There is a* 1*-local distributed approximation algorithm for multicoloring hexagonal graphs without a special heavy double-corner which uses at most* $\left\lceil \frac{4}{3}\omega(G) \right\rceil + O(1)$ *colors. The time complexity of the algorithm at each vertex is constant.*

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