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A survey-based approach for designing the lines of a rapid transit network

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ABSTRACT

In this work we present an approach for designing the lines of a rapid transit network. Given the stations to be constructed and the links between them, a set of lines is generated by utilizing a greedy heuristic procedure that, taking into account the transfers that should be made by the users to arrive at their destinations, attempts to maximize an estimation for the number of trips through the rapid transit network. This estimation is done by means of a modification of the well-known Floyd–Warshall algorithm. The main contributions are the consideration of the results from a survey amongst the potential users of the rapid transit network, and the contemplation of the possibility of linking certain pairs of station locations by more than one line. We also report some computational experience on several randomly generated instances that shows that our approach can obtain better line designs and handle instances of larger size than some other procedures taken from the literature.

1. Introduction

This paper presents a greedy heuristic procedure for generating the set of lines for a rapid transit network. It is the second and last stage of a new approach for designing rapid transit networks by making use of the results from a survey amongst the potential users. The first stage of this approach was stated in Escudero and Muñoz [4], and basically consists in selecting which stations and links to construct without exceeding the available budget, to maximize an estimation of the number of trips through the rapid transit network. Escudero and Muñoz [2] contains a preliminary version of the whole approach. We consider the same notation and assumptions as in [4].

It is worth noting that our greedy heuristic procedure could also be applied even if the stations and links to be constructed had not been obtained by the method provided in [4], as long as the associated rapid transit network is connected and its construction cost does not exceed the available budget. Consequently, our procedure could also be employed for redesigning the lines of existing rapid transit networks.

We consider two types of stations: key stations and non-key stations. The key station locations will be represented as the nodes of a graph, and the links between them as the edges of that graph. The non-key stations can be located on those links, and it is not necessary to represent them explicitly in the graph (see [4] for more details).

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We propose to solve the line design problem by means of a generalization of the greedy heuristic procedure given in Escudero and Muñoz [3] to allow pairs of station locations linked by more than one line, in such a way that the number of lines that go to each key station location is as small as possible, and attempting to maximize an estimation of the total number of trips through the rapid transit network. For this purpose, we introduce a modification of the well-known Floyd–Warshall algorithm to determine the shortest route between each pair of key station locations, and we define an expanded network that will make it possible to take into account the transfer times for the users.

The greedy heuristic procedure presented in Escudero and Muñoz [3] is in turn a modification of the one presented in Escudero and Muñoz [1]. Neither of these papers proposes to perform a survey amongst the potential users of the rapid transit network, and both of them consider only one type of station, assume that whichever two station locations are linked by one line at most, and impose that the number of lines that go to each station location is as small as possible. However, [1] does not take into account the transfers that should be made by the users to arrive at their destinations, whereas [3] does, attempting to minimize an estimation of the total number of such transfers.

An extensive literature review on line planning in public transportation is given in Schöbel [8].

The greedy heuristic procedure proposed in this paper can be outlined as follows: The generalization of the greedy heuristic algorithm given in Escudero and Muñoz [3] for minimizing an estimation of the total number of transfers through the rapid transit network is applied to the graph that represents the stations and links to be constructed (this estimation is computed from the shortest routes in the associated network between all the pairs of key station locations). Thus, a set of lines is obtained such that whichever two key station locations are linked by one line at most. Then, an estimation for the total number of trips through the rapid transit network is computed from the shortest routes in the associated expanded network between all the pairs of key station locations. This approach is repeated iteratively attempting to obtain other sets of lines that, without exceeding the available budget for constructing the rapid transit network, increase the value of the estimation for the total number of trips and allow pairs of key station locations linked by δ lines at most, where δ initially takes the value 2 and is sequentially increased by 1 until it is not possible to determine a set of lines better than the best set of lines so far obtained.

The remainder of the paper is organized as follows: Section 2 summarizes the main notation and assumptions that we consider. Section 3 presents a modification of the Floyd–Warshall algorithm to determine a shortest chain between each pair of nodes of a graph with nonnegative length edges. Section 4 shows how to calculate the maximum possible expected number of trips taken on the rapid transit network between pairs of key station locations, by applying the modification of the Floyd–Warshall algorithm to determine the shortest chains in a certain network, assuming that no transfers are required as well as that the capacity of the rapid transit network is enough to hold all those trips. Section 5 provides an algorithm for designing a set of lines for the rapid transit network which generalizes the greedy heuristic algorithm given in Escudero and Muñoz [3], to allow pairs of station locations linked by more than one line. Section 6 shows how to calculate in a more accurate way the maximum possible expected number of trips obtained in Section 4, by applying the modification of the Floyd–Warshall algorithm to determine the shortest chains in an expansion of the network considered therein that allows to take into account the transfer times. Section 7 proposes a greedy heuristic procedure for determining a line design for the rapid transit network, attempting to maximize the expected total number of trips through the rapid transit network. Section 8 reports some computational experience on the same example cases considered in Escudero and Muñoz [4]. Finally, Section 9 draws some conclusions and future research from this work.

2. Notation and assumptions

Below we provide a summarized list containing the main notation and assumptions considered in Escudero and Muñoz [4], in order of appearance (see [4] for more details).

V	set of key station locations ($V = \{1, \ldots, n\}$)
E	set of (nonordered) pairs of key station locations that can potentially be linked ($E = \{\{i, j\} \in V \times V \mid i < j$ and it is possible to link i and $j\}$)
$m = \lvert E \rvert$	
$G = (V, E)$	(G is a simple connected graph)
a_i	cost of constructing a key station at location i
$\Gamma(i)$	set of key station locations that can be linked to location i ($\Gamma(i)$ is the set of nodes adjacent to i in G)
d_{ij}	length (in km.) of link $\{i, j\}$
s_{ij}	number of non-key station locations on link $\{i, j\}$
c_{ij}	cost of linking i and j (including the cost of constructing the corresponding non-key stations)
b	available budget for constructing the rapid transit network
$\overline{v}$	average velocity (in km/h) of the network's vehicles
$\overline{t}(i)$	average time (in min.) required for going between the entrance of the key station located at i and its boarding and alighting platform
$\overline{t}_a$	average interarrival time (in min.) of the vehicles at each station