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Approximate robust optimization for the Connected Facility Location problem

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ABSTRACT

In this paper we consider the Robust Connected Facility Location (ConFL) problem within the robust discrete optimization framework introduced by Bertsimas and Sim (2003). We propose an Approximate Robust Optimization (ARO) method that uses a heuristic and a lower bounding mechanism to rapidly find high-quality solutions. The use of a heuristic and a lower bounding mechanism – as opposed to solving the robust optimization (RO) problem exactly – within this ARO approach significantly decreases its computational time. This enables one to obtain high quality solutions to large-scale robust optimization problems and thus broadens the scope and applicability of robust optimization (from a computational perspective) to other NP-hard problems. Our computational results attest to the efficacy of the approach.

1. Introduction

The Connected Facility Location (ConFL) problem models a variety of problems that are significant in the telecommunications and data management literature (see [14,17]), as well as the emergency management literature (see [26]). By design the ConFL problem is deterministic; however, the practical settings that motivate it are characterized by significant uncertainty. In this paper we make the first attempt to address this uncertainty in the ConFL problem using robust optimization (RO) (within the RO framework introduced by [7] for discrete optimization problems) and provide a heuristic approach that yields high-quality solutions.

The ConFL problem encompasses a large family of network design problems [3], where at minimum cost a set of facilities must be opened, customers must be assigned to open facilities, and lastly, open facilities must be connected through a Steiner tree. In this setting, the main source of uncertainty lies in the assignment costs as the location and demand of customers are often unknown. Furthermore, there might be limited information about the distribution of those costs making it difficult to follow a traditional 2-stage stochastic programming approach to address uncertainty in the problem. However, if one can estimate a range for the costs (i.e., a lower bound and an upper bound), one can search for a *robust solution*.

One approach in robust optimization is to protect against the worst-case scenario. In other words the decision maker wants to find a solution to the problem that minimizes the overall cost for the *worst case* scenario. However, since this may be viewed as an ultraconservative approach (which can lead to an expensive solution), Bertsimas and Sim [7] (BS) propose an alternative RO approach that allows the decision maker to adjust the level of conservatism for the solution. Roughly speaking, in the BS approach instead of optimizing for the worst-case scenario, the goal is to find a solution that protects the

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decision-maker against all cases in which up to Γ parameters, instead of all of the uncertain parameters, take their worst value (the remaining parameters take their best case values). When Γ is equal to the number of uncertain parameters in the solution it corresponds to the worst-case scenario solution.

The BS approach thus deals with discrete optimization problems with interval uncertainty in the objective function coefficients. It requires a large but polynomial number of *nominal problems* to be solved. The nominal problems are (suitably modified) deterministic variations of the original problem. Consequently, when the nominal problem is polynomially solvable the robust problem is also polynomially solvable (which is a particularly nice feature of the approach). On the flip side, if the original problem is NP-hard, the nominal problems are also NP-hard and computationally expensive to solve to optimality. Thus, there are some significant computational challenges in applying the BS approach to NP-hard robust optimization problems. Alternatively, in this paper, we demonstrate how to use a heuristic and lower bound mechanism to solve the nominal problems approximately and yet obtain high-quality solutions for the robust problem. This is a significant computational advantage, as it makes it practical to apply the BS robust optimization paradigm to a large class of NP-hard optimization problems.

Bertsimas and Sim [7] discuss the application of approximation algorithms in the context of robust optimization. They show that an α-approximation algorithm for the nominal problem results in an α-approximation algorithm for the RO problem. Our work expands the scope of their result in a *computational practical manner*. Approximation algorithm ratios are worst-case results and they do not (generally) provide tailored lower bounds for specific problem instances. Further, many NP-hard problems are max-SNP hard (or inapproximable unless $P = NP$) and thus constant approximation ratios are not known for them. In a practical setting then the approximation algorithm may be somewhat dissatisfying when trying to obtain near-optimal solutions to the problem at hand. In the realm where one has access to a (good) heuristic and a (good) lower bounding mechanism for individual problem instances (which is often the case for hard combinatorial optimization problems), we show how to expand upon Bertsimas and Sim [7] to develop an approximate robust optimization procedure that provides both a heuristic solution and an overall lower bound for the RO problem. We illustrate this approach on the robust ConFL problem.

The rest of this paper is organized as follows. In Section 2 we introduce the robust ConFL problem and related literature. In Section 3 we review Bertsimas and Sim [7]'s robust optimization approach for discrete optimization problems with interval uncertainty in the objective function coefficients, and then present our Approximate Robust Optimization (ARO) method. In Section 4 we apply the ARO method to the robust ConFL problem. We also illustrate the ARO method on a small robust ConFL problem. We also consider a special case of the robust ConFL problem with disk uncertainty areas. In Section 5 we illustrate the effectiveness of the ARO method with extensive computational experiments on the robust ConFL problem. Section 6 provides concluding remarks.

2. Problem definition and related literature

We start by defining the ConFL problem, and later, expanding its definition to describe the robust ConFL problem considered in this paper.

2.1. Connected facility location problem

The ConFL problem can be stated as follows. We are given a graph $G = (V, E)$, and three disjoint sets: $D \subseteq V$, set of demand nodes (or customers); $F \subseteq V$, set of potential facility nodes; and $S \subseteq V$, set of potential Steiner nodes, with $D \cup F \cup S = V$. We seek to find a minimum cost network where every demand node is assigned to an open facility and open facilities are connected through a Steiner tree T constructed on the subgraph of G on the nodes $F \cup S$ (i.e., $G(F \cup S) = (F \cup S, E(F \cup S))$). There are facility opening costs, $f_i \geq 0$, incurred for each facility; assignment costs, $a_{ij} \geq 0$, for assigning a customer $j \in D$ to a facility $i \in F$; and edge costs, $b_{ij} \geq 0$, for an edge $\{i, j\} \in E(F \cup S)$ if it is used on the Steiner tree T. The nodes in S may be viewed as pure Steiner nodes and can only be used in the tree T as Steiner nodes, while the nodes in F may be used as Steiner nodes on the tree T incurring a facility opening cost even when no customers are assigned to them.[1] The final network cost is given by $\sum_{i\in Z} f_i + \sum_{\{i,j\}\in E(T)} b_{ij} + \sum_{j\in D} a_{i(j)j}$, where $i(j)$ is the facility serving demand node j, Z is the set of open facilities, and T is a Steiner tree connecting the open facilities. We first describe a formulation for the ConFL problem.

$$\text{Minimize} \sum_{i\in F} f_i z_i + \sum_{\{i,j\}\in E(S\cup F)} b_{ij} y_{ij} + \sum_{i\in F, j\in D} a_{ij} x_{ij} \tag{1a}$$

subject to

$$\sum_{\{i,j\}\in E(R)} y_{ij} \leq \sum_{l\in R\setminus k} z_l, \quad \forall R \subset (S \cup F),\ |R| \geq 3,\ \forall k \in R \tag{1b}$$

1 Our definition for the ConFL problem follows Bardossy and Raghavan [3]. They show this general definition of the ConFL captures other variants where the sets D, F, S overlap; or where facilities incur a cost only when customers are served from that facility. Consequently, this general definition includes all known variants of the ConFL problem as well as the Steiner tree star problem [16], and the rent or buy problem [12].