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Efficient k-shot broadcasting in radio networks☆

Erez Kantor [a,*], David Peleg [b]

[a] CSAIL MIT, Cambridge, MA, United States

[b] Weizmann Institute of Science, Rehovot, Israel

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ABSTRACT

The paper concerns time-efficient k-shot broadcasting in undirected radio networks for n-node graphs of diameter D. In a *k-shot* broadcasting algorithm, each node in the network is allowed to transmit at most k times. Both known and unknown topology models are considered. For the known topology model, the problem has been studied before by Gąsieniec et al. (2008).

We improve both the upper and the lower bound of that paper providing a randomized algorithm for constructing a k-shot broadcasting schedule of length $D+O(kn^{1/2k}\log^{2+1/k} n)$ on undirected graphs, and a lower bound of $D+\Omega(k\cdot(n-D)^{1/2k})$, which almost closes the gap between these bounds. For the unknown topology model, we provide the first k-shot broadcasting algorithm.

Assuming that each node knows only the network size n (or a linear upper bound on it), our randomized k-shot broadcasting algorithm completes broadcasting in $O((D+\min\{D\cdot k, \log n\})\cdot n^{1/(k-1)}\log n)$ rounds with high probability for $k\geq 2$, and in $O(D\cdot n^2\log n)$ rounds with high probability for $k=1$. Moreover, we present an $\Theta(\log n)$-shot broadcasting algorithm that completes broadcasting in at most $O(D\log n+\log^2 n)$ rounds with high probability. This algorithm matches the broadcasting time of the algorithm of Bar-Yehuda et al. (1992), which assumes no limitation on the maximum number of transmissions per node.

1. Introduction

In this paper we study the fundamental task of *broadcasting* in synchronous radio networks, in both the known and unknown topology models. A radio network consists of *stations* that can act, at any given time step (round), as either a transmitter or a receiver. The network is modeled as an undirected graph $G(V,E)$, where V represents the set of stations and E represents communication feasibility, i.e., two nodes $u,v\in V$ can communicate directly with each other iff $(u,v)\in E$.

Energy efficiency is a central issue in designing the operation of ad-hoc radio networks and sensor networks, as in many cases the only energy sources for the stations are limited lifetime batteries. This paper concerns the use of k-shot algorithms, where each node in the network is allowed to transmit at most k times, hence energy is preserved at each of the stations. Such a strategy for energy efficient radio communication was studied in the context of broadcasting and gossiping in radio networks of random topology, see [4,13].

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* Corresponding author.
E-mail addresses: erezk@csail.mit.edu (E. Kantor), david.peleg@weizmann.ac.il (D. Peleg).

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In the unknown topology model, we assume that each node knows only a linear upper bound on the number of nodes n, but does not know anything else concerning the topology. This model is often used to describe *sensor* networks and is particularly suitable for *ad hoc* networks. A sensor network is composed of a large number of sensor nodes, which can be densely deployed in the targeted environment, and communicate via an ad hoc wireless network. Sensor networks are expected to take part in many civil and military applications, for example, in earthquake and tsunami warnings, fire detection, etc.

These networks possess some unique features, which introduce also new algorithmic challenges. The sensor devices are very cheap, they are prone to failures, and the number of sensors in a network is generally very large. The battery's life time in each unit is limited, hence energy saving is an acute concern. In many cases, the deployment of sensor networks is such that the positions of the sensors are unknown. For example, in certain cases the sensors are spread out from the air by an airplane; in such cases the topology of the network is unknown. On the other hand, in many other settings the network topology is known, and the stations may either know the topology in advance or can learn the topology during some preprocessing procedure. We study this model as well, and present an algorithm that computes a fast broadcasting scheduler under this model.

We consider a synchronous network, where communication is performed in rounds and is assumed to have the following property: (1) in each round a node either acts as a transmitter, or does not act as a transmitter (in which case we say that it acts as a receiver); (2) a message transmitted by a station reaches all its neighbors; and (3) a node $u \in V$ *receives* a message M in a given round if and only if on that round it does not act as a transmitter and exactly one of its neighbors acts as a transmitter and transmits M. Otherwise (in case u does not act as a transmitter), there are two possibilities: if none of u's neighbors transmits, then u hears *silence*, and if at least two of u's neighbors transmit simultaneously, then a *collision* occurs at u. In both cases, u does not receive any message.

We consider *broadcasting*, which is the following communication task. A distinguished node s, called the *source*, has a message M that has to be delivered to all other nodes in the network. A broadcasting schedule S in a radio network is a list $(T_1, T_2, \ldots, T_t)$ of subsets of V that describes the order of transmissions: for each round $i = 1, 2, \ldots, t$, the set $T_i \subseteq V$ specifies the nodes that have to act as transmitters on round i. We assume that a node v scheduled to act as a transmitter on round t will transmit the source message M if it has already received it from one of its neighbors in some previous round. The *length* of the schedule S is the number of rounds, t, and S is said to complete broadcasting if by time t, all the network nodes have received M.

Our contribution: We study the k-shot broadcasting in undirected radio networks. Both the known and unknown topology model are considered. For the known topology model, the problem has been studied before by Gąsieniec et al. [13]. That paper presented a deterministic 1-shot broadcasting protocol that completes the broadcasting task in $D + O(\sqrt{n}\log n)$ rounds, and a randomized k-shot broadcasting protocol for $k \geq 3$, that completes the broadcasting task in $D + O(kn^{1/(k-2)}\log^2 n)$ rounds with high probability. In addition, the authors proved a lower bound of $D + \Omega((n-D)^{1/2k})$ on the length of k-shot broadcasting schedules for n-node graphs of diameter D. We improve both the upper and the lower bound. Specifically, in Section 3 we present a randomized algorithm for constructing a k-shot broadcasting schedule of length $D + O(kn^{1/2k}\log^{2+1/k} n)$ on undirected graphs, which almost matches the lower bound. For the lower bound we show that on binomial bipartite graphs, presented in [13] (see Section 3.5), any broadcasting schedule requires at least $\Omega(k \cdot n^{1/2k})$ rounds, implying a lower bound of $D + \Omega(k \cdot (n-D)^{1/2k})$ rounds on arbitrary undirected graphs.

For the unknown topology model, we present in Section 2 a first k-shot broadcasting algorithm. Assuming that each node knows only the network size n (or a linear upper bound on it), our randomized k-shot broadcasting algorithm completes broadcasting in $O((D + \min\{D \cdot k, \log n\}) \cdot n^{1/(k-1)}\log n)$ rounds with high probability for $k \geq 2$, and in $O(D \cdot n^2 \log n)$ rounds with high probability for $k = 1$.

Moreover, we present a $\Theta(\log n)$-shot broadcasting algorithm that completes broadcasting in at most $O(D \cdot \log n + \log^2 n)$ rounds with high probability. This algorithm is competitive with the expected $O(\log^2 n/\log(n/D))$-shots randomized broadcasting algorithm with optimal broadcasting time of $O(D\log(n/D) + \log^2 n)$ presented by Berenbrink et al. [4], and matches the broadcasting time of the algorithm of Bar-Yehuda et al. [3], which assumes no limitation on the maximum number of transmissions per node (and is, in effect, an $O(\log^2 n)$-shot broadcasting algorithm using expected $O(\log n)$-shots per node). (Note that the broadcasting algorithm of Berenbrink et al. [4] and the broadcasting algorithms of Czumaj and Rytter [10] and Kowalski and Pelc [18] all use $O(\log^2 n)$ shots per node, i.e., all are $O(\log^2 n)$-shot broadcasting algorithms.) A comparative summary of these results is provided in Table 1.

Related work: Deterministic centralized broadcasting in radio networks was first studied by Chlamtac and Kutten [5], who formulated the radio network model. A lower bound of $\Omega(\log^2 n)$ time for broadcasting, even in $O(1)$-diameter networks, was established in [1] by showing the existence of a family of radius-2 n-node networks for which any broadcast schedule requires at least $\Omega(\log^2 n)$ rounds. On the other hand, for the known topology model, a sequence of papers presented increasingly tighter upper bounds. In [6], Chlamtac and Weinstein presented an $O(D\log^2 n)$-time broadcasting algorithm for n-node radio networks of diameter D. In [12], Gaber and Mansour proposed an $O(D + \log^5 n)$-time broadcasting algorithm. Subsequently, Elkin and Kortsarz [11] presented a deterministic algorithm yielding schedules of length $O(D + \log^4 n)$, Gąsieniec et al. [14] presented a deterministic algorithm for constructing schedules of length $D + O(\log^3 n)$ and a randomized algorithm for computing schedules of length $D + O(\log^2 n)$, and finally Kowalski and Pelc [20] gave an optimal deterministic algorithm yielding schedules of $O(D + \log^2 n)$ rounds.